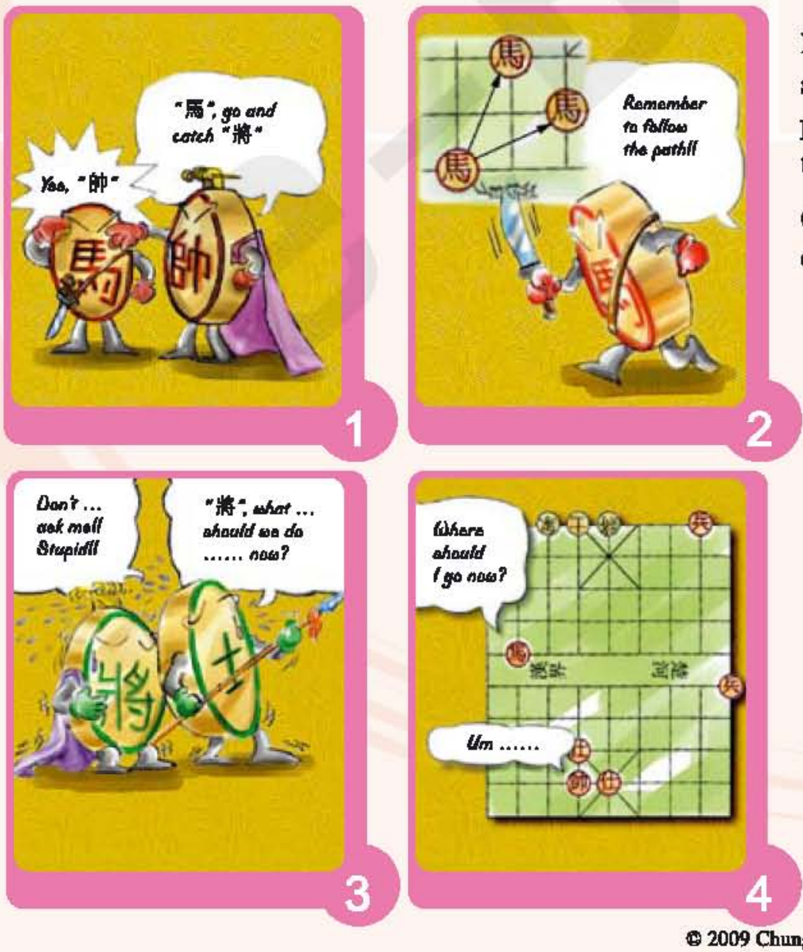
Chapter 9

Introduction to Coordinates

Learning Objectives

After completing this chapter, you will be able to

- understand and use rectangular coordinates and polar coordinates to describe the positions of points on a plane.
- use ordered pairs of a rectangular coordinate system to locate points on a plane.
- find the distance between two points on a horizontal or vertical line on a rectangular coordinate plane.
- find the areas of polygons on a rectangular coordinate plane.
- describe the effects of transformations on the points on a coordinate plane intuitively.



In this chess game, design a simple system to describe a path for the piece 馬 to catch the piece 將.

(The piece 馬 must move diagonally in a 2 × 1 rectangle.)

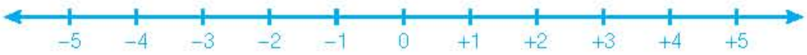
Preview

[Basic knowledge and techniques required for this chapter.]

A. Basic Knowledge

Number line

Positive and negative numbers can be represented by points on a straight line. This straight line is called the number line.



B. Basic Technique

1. Comparison between two directed numbers

Example: **(a)** $7 > 2$ **(b)** $-3 < 2$

(c) $1 > -4$ **(d)** $-9 < -4$

2. Subtraction of directed numbers

Example: **(a)** $10 - 3 = 7$ **(b)** $10 - (-3) = 13$

(c) $-10 - 3 = -13$ **(d)** $-10 - (-3) = -7$

9.1 Ordered Pairs

Class Activity 9.1

Aim: To understand the concept of an ordered pair

1. Figure I shows the first 4 rows of seats in a mini-cinema. Alan's seat number is A5, which represents the position X in the figure. The positions of Suki, Bonnie and Cathy are Y, Z and W respectively.

Show Ans.

(a) Suki's seat number: __________

Bonnie's seat number: __________

Cathy's seat number: __________

(b) If Joseph also watches the movie and sits beside Cathy, his seat number is __________.

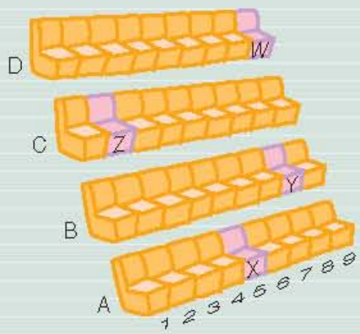


Figure I

2. Figure II is the seating plan of a classroom. Show Ans.

(a) Suppose Kelvin sits in the 2nd row of the 1st column, describe the position of Charlie in terms of row and column.

____

(b) Alfred sits next to Charlie. Try to describe Alfred's position in terms of row and column.

____

(c) Where does Derek sit?

____

(d) Who sits in the 3rd row of the 2nd column?

____

(e) Who sits in the 2nd row of the 3rd column?

____

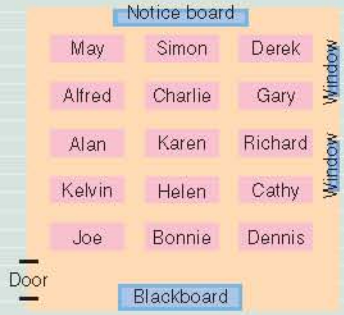


Figure II

3. Map reading

Figure III shows the map of Hong Kong Island on a piece of graph paper. In order to show the position of a place, we have to use column and row, e.g. the position of Central can be represented by column 4 and row 8, or simply (4, 8). This pair of numbers arranged in a special order within brackets is called an **ordered pair**. We can represent positions of any places on the map using ordered pairs.

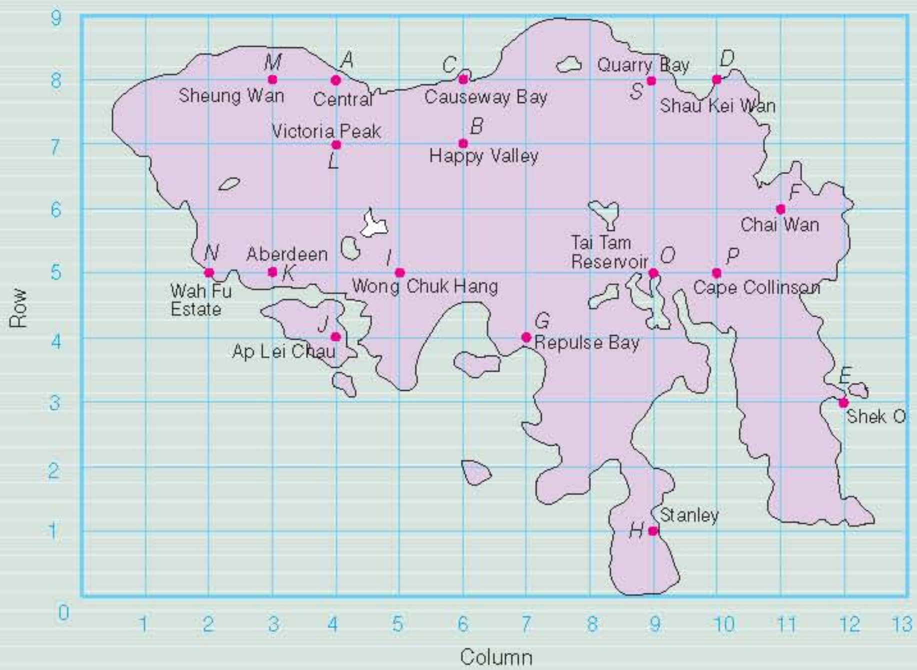


Figure III

ordered pair 序偶

Show Ans.

(a) Complete the following. [**(a)(i)** is given as an example.]

(i) (4, 7) representing the position L is Victoria Peak.

(ii) (5, 5) representing the position ______ is ________________.

(iii) (2, 5) representing the position ______ is ________________.

(iv) (,   ) representing the position ______ is Sheung Wan.

(v) (,   ) representing the position ______ is Tai Tam Reservoir.

(vi) (,   ) representing the position P is ________________.

(b) Write down the ordered pairs of the following positions.

A (4, 8) B(,   ) C(,   ) D(,   )

E(,   ) F(,   ) G(,   ) H(,   )

(c) Do ordered pairs (4, 7) and (7, 4) represent the same place? Why?

____

____

(d) What is common among the ordered pairs representing Sheung Wan, Central, Causeway Bay and Quarry Bay?

____

(e) What is common among the ordered pairs representing Quarry Bay, Tai Tam Reservoir and Stanley?

____

Now I see ...

We can use an ordered pair to represent the position of a point on a plane.

In the Class Activity, three different ways are used to represent the position of a point on a plane:

In question **1**, 'A5' represents the position that Alan sits.

In question **2**, '2nd row of 1st column' represents the position that Kelvin sits.

In question **3**, (4, 8) represents the position of Central.

Representing the position of a point by an ordered pair (a, b) is often used in Mathematics. One should pay attention to the proper order of numbers when writing an ordered pair, otherwise the point may be mistakenly located. For example, (4, 7) and (7, 4) in question **3** represent Victoria Peak and Repulse Bay respectively.

9.2 Rectangular Coordinate System

In this section, we will study a system using ordered pairs to represent the positions of points on a plane, called the **rectangular coordinate system**.

A Rectangular coordinate plane

I. Coordinates

Figure 9.1 shows a **rectangular coordinate plane**. It is formed by a horizontal line (***x*-axis**) and a vertical line (***y*-axis**).

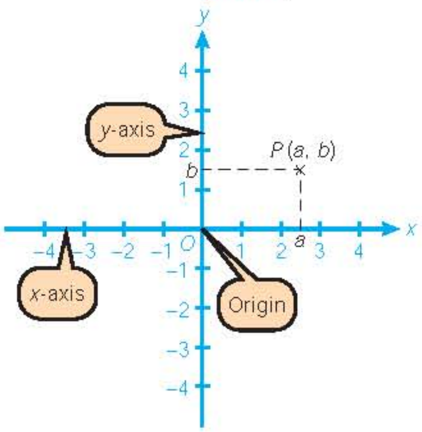


Figure 9.1

Of which,

1. the x-axis and y-axis are called the **coordinate axes**, their point of intersection (i.e. point O) is called the **origin**.
2. the position of any point P on the plane can be represented by the ordered pair (a, b). a and b are the ***x*-coordinate** and ***y*-coordinate** of the point respectively. (a, b) are also called the **coordinates** of P. P can also be represented by $P(a, b)$.

◂ The coordinates of the origin are (0, 0).

We add grid lines onto the plane for easier reading. Figure 9.2 shows the positions of some points on a rectangular coordinate plane.

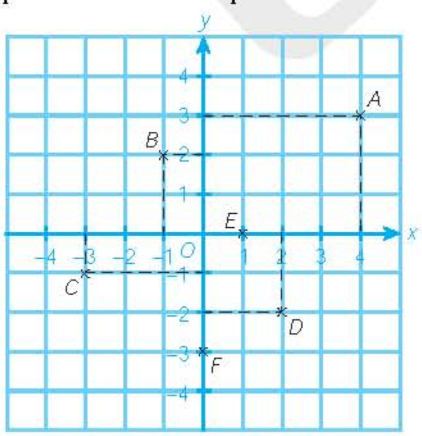


Figure 9.2

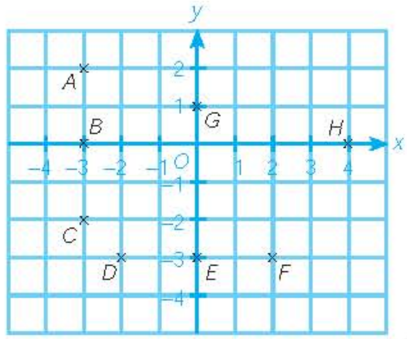
The coordinates of point A are $(4, 3)$;
the coordinates of point B are $(-1, 2)$;
the coordinates of point C are $(-3, -1)$;
the coordinates of point D are $(2, -2)$;
the coordinates of point E are $(1, 0)$; and
the coordinates of point F are $(0, -3)$.

rectangular coordinate system 直角坐標系統
rectangular coordinate plane 直角坐標平面
x-axis x 軸
y-axis y 軸
coordinate axis 坐標軸
origin 原點
x-coordinate x 坐標
y-coordinate y 坐標
coordinates 坐標

Additional Example 9.1

Example 9.1 Writing down the coordinates of points

The figure shows the positions of points A to H on a rectangular coordinate plane.



(a) Write down the coordinates of the points in the figure.

(b) Which points are on the x-axis? What are their y-coordinates?

(c) Which points are on the y-axis? What are their x-coordinates?

Solution

(a) $A(-3, 2)$, $B(-3, 0)$, $C(-3, -2)$, $D(-2, -3)$, $E(0, -3)$, $F(2, -3)$, $G(0, 1)$, $H(4, 0)$

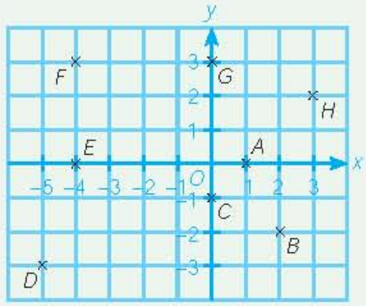
(b) B and H are on the x-axis. Their y-coordinates are both 0.

(c) E and G are on the y-axis. Their x-coordinates are both 0.

Solution

Classwork 9.1

The figure shows the positions of points A to H on a rectangular coordinate plane.



(a) Write down the coordinates of the points in the figure.

(b) Which points have the x-coordinates equal to 0?

(c) Which points have the y-coordinates equal to 0?

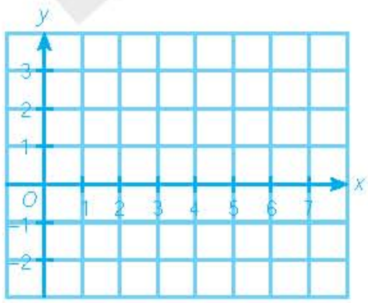
Additional Example 9.2

Example 9.2 Marking the points on a rectangular coordinate plane

(a) Mark the points $A(6, -2)$, $B(6, 0)$ and $C(6, 3)$ on the rectangular coordinate plane.

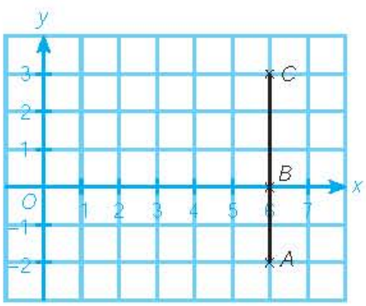
(b) Join A, B and C together.

(c) Which coordinate axis is parallel to the line segment AC?



Solution

(a), (b)

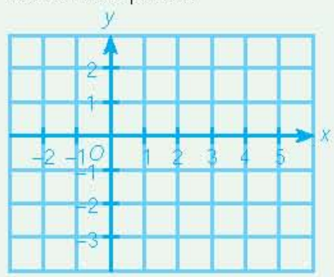


(c) The y-axis is parallel to the line segment AC.

Solution

Classwork 9.2

(a) Mark the points $A(5, -2)$, $B(0, -2)$ and $C(-2, -2)$ on the rectangular coordinate plane.



(b) Join A, B and C together.

(c) Which coordinate axis is parallel to the line segment AC?

II. Quadrant

In a rectangular coordinate plane, the x-axis and y-axis divide the plane into 4 regions. Each of these regions is called a **quadrant**.

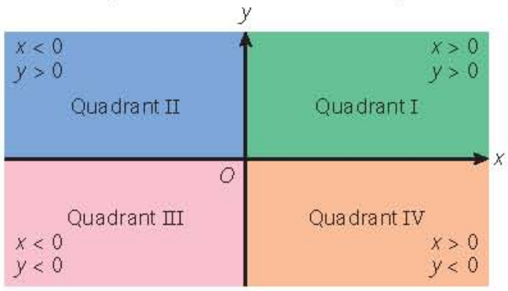
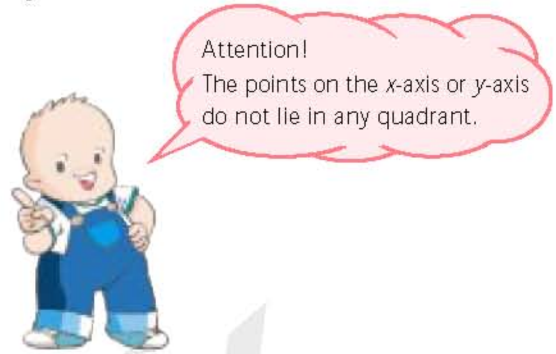


Figure 9.3



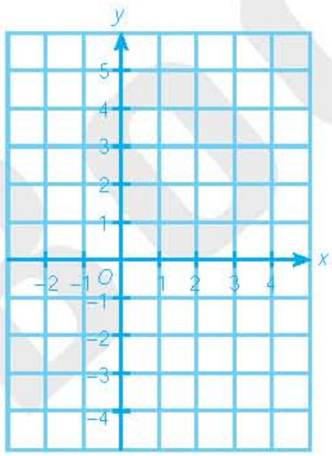
Additional Example 9.3

Example 9.3 Determining the points lying in different quadrants

(a) Mark the following points on the rectangular coordinate plane.
$A(1, 5)$, $B(-2, 3)$, $C(4, 3)$, $D(1, 2)$, $E(-2, 0)$, $F(4, 0)$, $G(1, -2)$, $H(-1, -2)$, $I(-1, -4)$, $J(3, -4)$, $K(3, -2)$

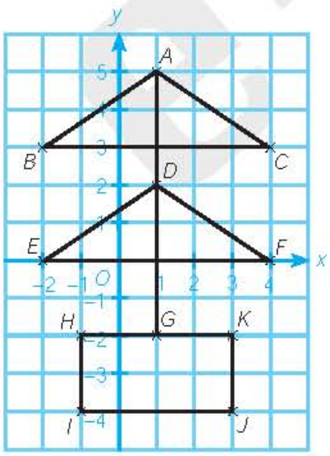
(b) Join the points according to the order A, B, C, A, D, E, F, D, G, H, I, J, K, G.

(c) Write down the points lying in each quadrant.



Solution

(a), (b)



(c) The points lying in quadrant **I** are A, C and D.

The point lying in quadrant **II** is B.

The points lying in quadrant **III** are H and I.

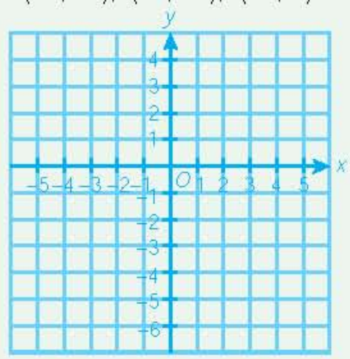
The points lying in quadrant **IV** are G, J and K.

quadrant 象限

Classwork 9.3

Solution

(a) Mark the following points on the rectangular coordinate plane.
$A(-4, 4)$, $B(0, 2)$, $C(4, 4)$, $D(3, 0)$, $E(5, -2)$, $F(2, -3)$, $G(0, -6)$, $H(-2, -3)$, $I(-5, -2)$, $J(-3, 0)$



(b) Join the points according to the order A, B, C, D, E, F, G, H, I, J, A.

(c) Write down the points lying in each quadrant.

B Drawing a rectangular coordinate plane on graph paper

The steps of drawing a rectangular coordinate plane on graph paper are as follows:

Step 1: On a piece of graph paper, draw two perpendicular lines. Label the horizontal line as the x-axis, the vertical line as the y-axis and their point of intersection as origin O.

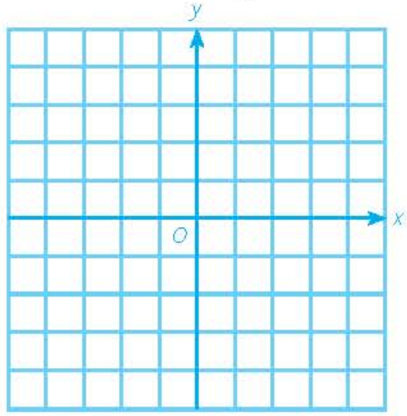


Figure 9.4

Step 2: Mark the scale numbers on each axis.

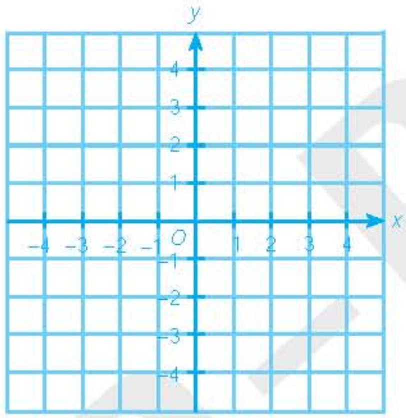


Figure 9.5

When drawing a rectangular coordinate plane, we should decide the scale on the coordinate axes and the position of the origin according to the given conditions. Sometimes, a rectangular coordinate plane may need to cover a wider range or to concentrate on a certain region.

The following are some examples:

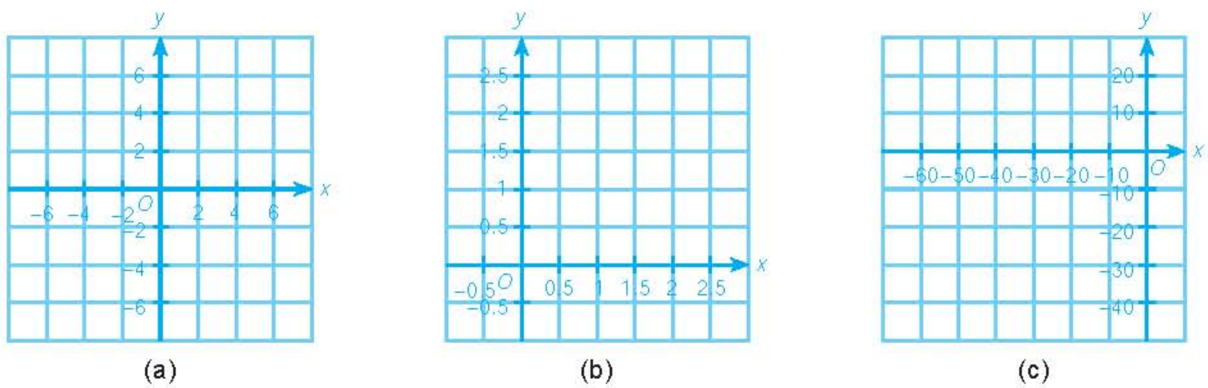


Figure 9.6

Additional Example 9.4

Example 9.4 Writing down the coordinates of the point of intersection

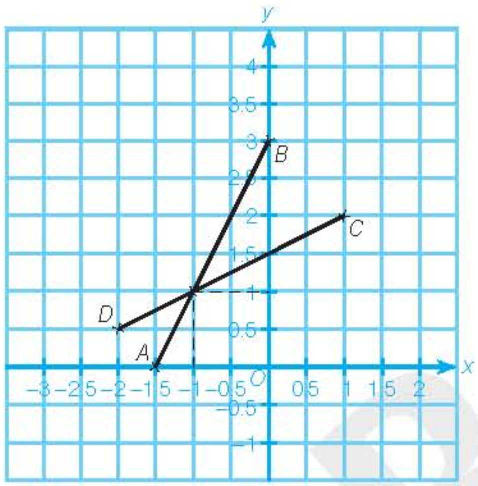
(a) Mark four points $A(-1.5, 0)$, $B(0, 3)$, $C(1, 2)$ and $D(-2, 0.5)$ on the rectangular coordinate plane.

(b) Join AB and join CD.

(c) Write down the coordinates of the point of intersection of AB and CD.

Solution

(a), (b)

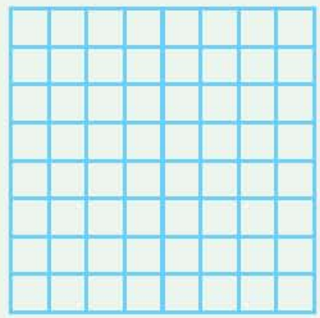


(c) From the figure, the coordinates of the point of intersection are $\underline{\underline{(-1, 1)}}$.

Classwork 9.4

Solution

(a) Mark the following points on the rectangular coordinate plane.
$P(-1, 1)$, $Q(0.5, -2)$, $R(-2, -1.5)$, $S(0.5, 1)$



(b) Join PQ and join RS.

(c) Write down the coordinates of the point of intersection of PQ and RS.

Skills Upgrading Corner 9.1

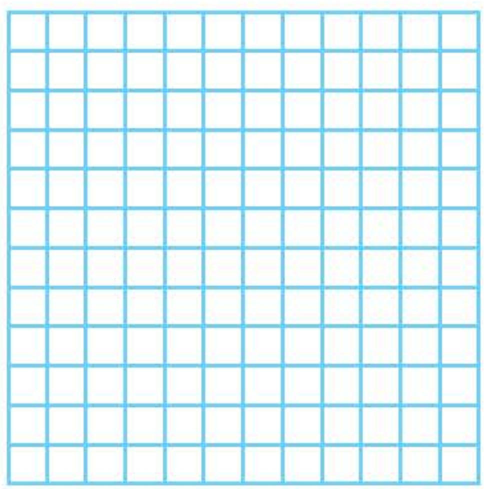
Solution

(a) Mark four points $A(3, 5)$, $B(-1, -3)$, $C(1, 4)$ and $D(6, -1)$ on the rectangular coordinate plane.

(b) Join AB and join CD.

(c) Write down the coordinates of the point of intersection of AB and CD.

(d) Write down the quadrant where the point of intersection of AB and CD lies in.

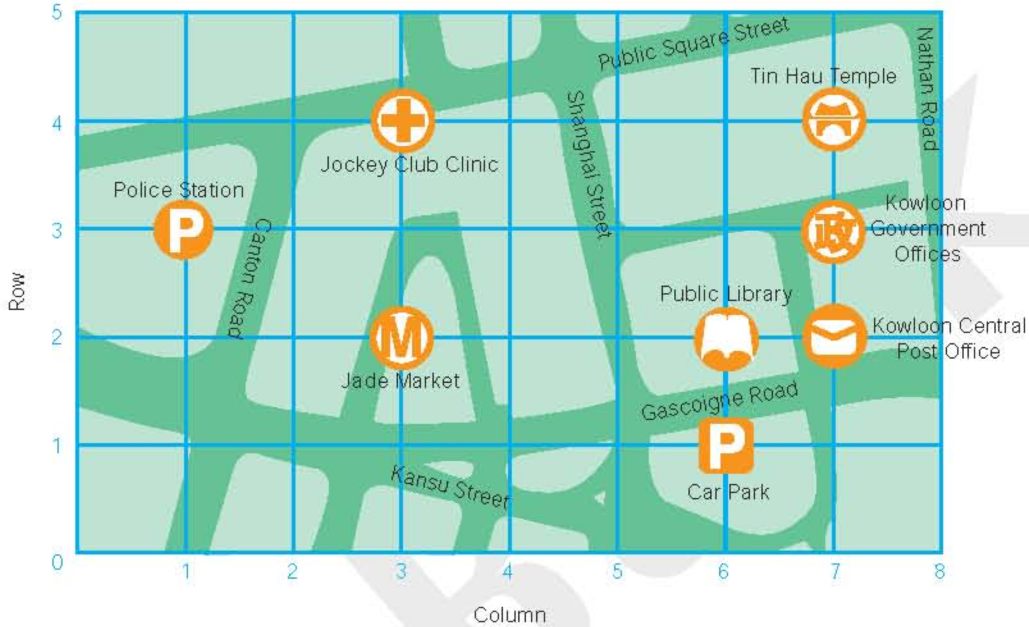


Exercise 9A

[Graph paper is provided in the Appendix.]

Level 1

1. The following is a map of Yau Ma Tei.

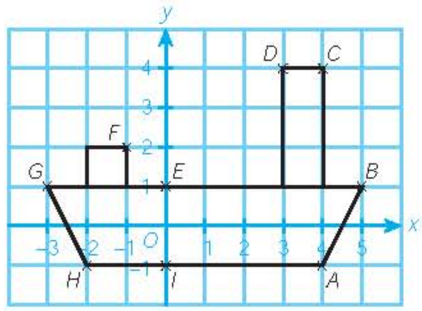


Represent the following locations using ordered pairs.

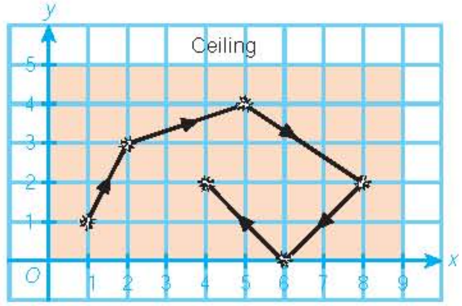
(a) Jockey Club Clinic **(b)** Kowloon Central Post Office **(c)** Tin Hau Temple

(d) Car park **(e)** Police station **(f)** Public library

2. The figure shows the positions of points A to I on a rectangular coordinate plane. Write down the coordinates of the points in the figure.



3. One morning, Kate laid on the bed and looked at the ceiling of her room. A fly came in the room and stopped at 6 different positions, Kate imagined the ceiling as a 9×5 rectangular grid, and worked out the route of the fly as shown in the figure. Write down the coordinates of the locations where the fly stopped.



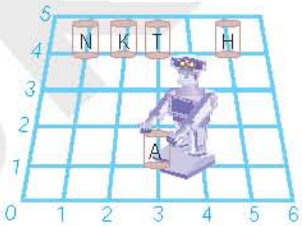
4. Write down the x-coordinates of the following points.
(a) $A(2, 6)$ (b) $B(-3, 4)$ (c) $C(0, 7)$

5. Write down the y-coordinates of the following points.
(a) $P(4, 5)$ (b) $Q(-3, 3)$ (c) $R(2, -2)$

6. Fill in the table with the points represented by the following coordinates.
$A(2, 5)$ $B(3, -2)$ $C(-4, 7)$ $D(0, -4)$ $E(-3, 0)$ $F(-2, -1)$
$G(-7, 9)$ $H(1, 0)$ $I(3, 4)$ $J(0, 5)$ $K(-6, -1)$ $L(2, -8)$

Quadrant I	Quadrant II	Quadrant III	Quadrant IV	x-axis	y-axis
A,					

7. Charles has made a robot which can move a letter from a specified position to a new position. The following table shows the original positions of the letters and their corresponding new positions.



Letter	N	K	T	A	H
Original position	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)
New position	(4, 1)	(5, 1)	(1, 1)	(3, 1)	(2, 1)

Write down the five letters according to the order of their new positions from left to right.

Level 2

8. (a) Mark three points $A(2, -2)$, $B(2, 1)$ and $C(2, 4)$ on a rectangular coordinate plane.
(b) Join A, B and C together.
(c) Which coordinate axis is parallel to the line segment AC?
(d) What is common among these coordinates?

9. (a) Mark three points $P(-2, 3)$, $Q(0, 3)$ and $R(1, 3)$ on a rectangular coordinate plane.
(b) Join P, Q and R together.
(c) Which coordinate axis is parallel to the line segment PR?
(d) What is common among these coordinates?

10. (a) Mark two points $P(2, 3)$ and $Q(-2, -3)$ on a rectangular coordinate plane.
(b) Join PQ.
(c) Does the line segment PQ pass through the origin?

11. (a) Mark four points $A(0, 1)$, $B(2, 5)$, $C(-2, -3)$ and $D(4, 4)$ on a rectangular coordinate plane.
(b) Which three points can form a straight line?

12. (a) Mark four points $A(4, 4)$, $B(2, 0)$, $C(6, 3)$ and $D(0, 1)$ on a rectangular coordinate plane.
(b) Join AB and join CD.
(c) Write down the coordinates of the point of intersection of AB and CD.

13. **(a)** Mark three points $A(3, 4)$, $B(3, 1)$ and $C(7, 1)$ on a rectangular coordinate plane.

(b) Measure $\angle ABC$.

14. **(a)** Mark four points $A(2, 1)$, $B(-2, 1)$, $C(-2, -3)$ and $D(2, -3)$ on a rectangular coordinate plane.

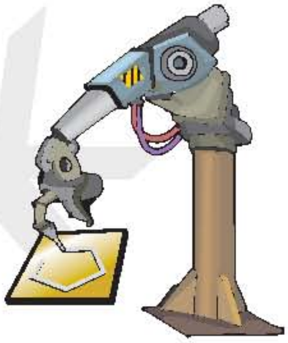
(b) Join the points according to the order A, B, C, D and A.

(c) What kind of quadrilateral is $ABCD$?

(d) Draw the two diagonals of the quadrilateral, and write down the coordinates of the point of intersection of the two diagonals.

(e) Write down the quadrant where the point of intersection of the two diagonals lies in.

15. A *mechanical arm* in a factory can perform cutting tasks through a computer program. If the coordinates of the vertices of a figure are input, the arm will cut the figure out accordingly. If the following coordinates are input, what figure will be obtained?



(a) $(1, 3)$, $(-2, 2)$, $(2, -1)$

(b) $(-4, 1)$, $(-1, 1)$, $(-1, 4)$, $(-4, 4)$

(c) $(0, 0)$, $(5, 0)$, $(6, 3)$, $(1, 3)$

(d) $(-2, -3)$, $(2, -3)$, $(1, 1)$, $(-2, 1)$

(e) $(0, 8)$, $(5, 0)$, $(0, -8)$, $(-10, -4)$, $(-10, 5)$

9.3 Distances

Class Activity 9.2

Aim: To explore the method of finding the distance of two points on a horizontal or vertical line on a rectangular coordinate plane

1. Refer to Figure I and answer the following questions. Show Ans.

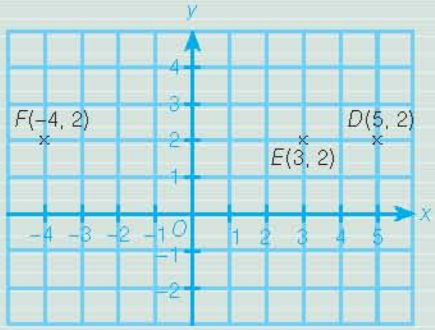


Figure I

(a) **(i)** Write down the three points in the figure according to the order of their locations from left to right.

____

(ii) Arrange the x-coordinates of D, E and F in ascending order.

____

(iii) If the x-coordinate of point K is larger than the x-coordinate of point L, then on which side of L does K lie?

____

mechanical arm 機械臂

(b) (i) Consider the x-coordinates of D and E, write down an expression for the length of DE.

____

(ii) Consider the x-coordinates of E and F, write down an expression for the length of EF.

____

(iii) If the x-coordinate (m) of point M is larger than the x-coordinate (n) of point N, and their y-coordinates are equal, write down an expression for the length of MN.

____

2. Refer to Figure II and answer the following questions. Show Ans.

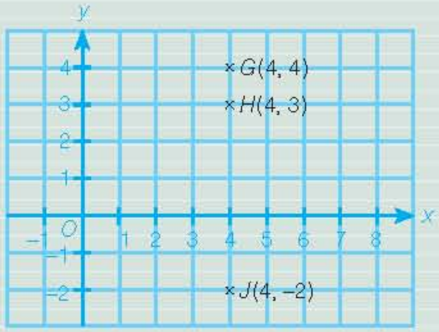


Figure II

(a) (i) Write down the three points in the figure according to the order of their locations from bottom to top.

____

(ii) Arrange the y-coordinates of G, H and J in ascending order.

____

(iii) If the y-coordinate of point P is larger than the y-coordinate of point Q, then on which side of Q does P lie?

____

(b) (i) Consider the y-coordinates of G and H, write down the expression for the length of GH.

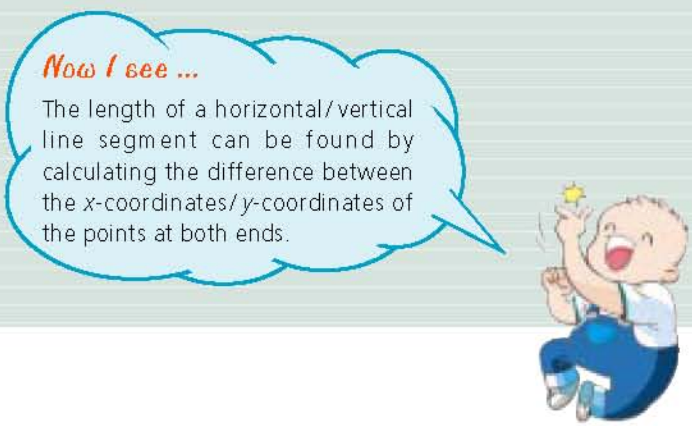
____

(ii) Consider the y-coordinates of H and J, write down the expression for the length of HJ.

____

(iii) If the y-coordinate (r) of point R is larger than the y-coordinate (s) of point S, and their x-coordinates are equal, write down an expression for the length of RS.

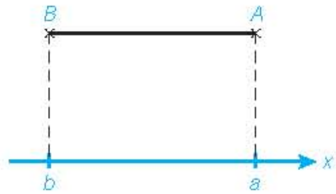
____



From the Class Activity,

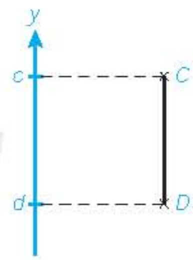
1. if A and B are two points on a horizontal line, A is on the right of B, and the x-coordinates of A and B are a and b respectively, then

$$\boxed{AB = a - b}$$



2. if C and D are two points on a vertical line, C is above D, and the y-coordinates of C and D are c and d respectively, then

$$\boxed{CD = c - d}$$



Extension 9.1

Solution

1. (a) What is common among the coordinates of points on a horizontal line?
   (b) What is common among the coordinates of points on a vertical line?

Solution

2. The figure shows some vertical and horizontal line segments. Find the lengths of all the line segments.

$AB =$ ____________

$CD =$ ____________

$EF =$ ____________

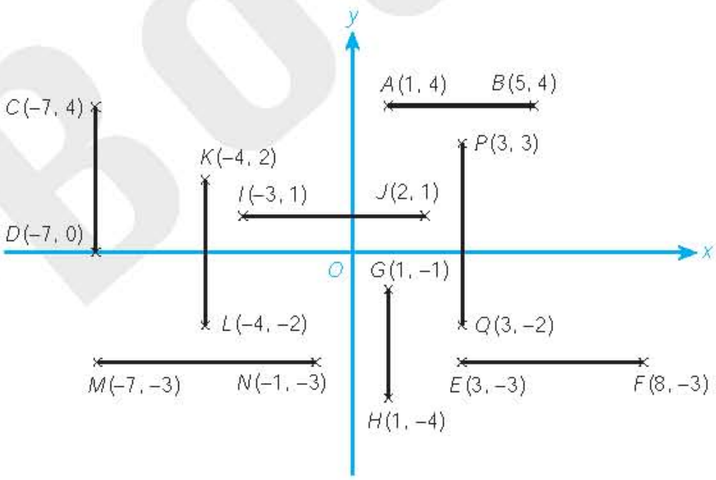
$GH =$ ____________

$IJ =$ ____________

$KL =$ ____________

$MN =$ ____________

$PQ =$ ____________



Additional Example 9.5

Example 9.5 Finding the distance between two points

Find the distance between each pair of points below.

(a) $A(3, 4)$, $B(2, 4)$

(b) $P(2, -1)$, $Q(2, 5)$

Solution

(a) $AB = (3 - 2)$ unit ◂ AB is a horizontal line segment

$= \underline{\underline{1 \text{ unit}}}$

(b) $PQ = [5 - (-1)]$ units ◂ PQ is a vertical line segment

$= \underline{\underline{6 \text{ units}}}$

Classwork 9.5

Solution

Find the distance between each pair of points below.

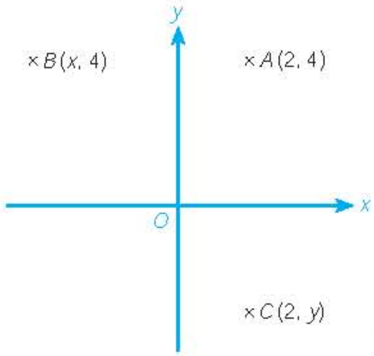
(a) $A(-4, 2)$, $B(3, 2)$

(b) $P(-7, -8)$, $Q(-7, -12)$

Additional Example 9.6

Example 9.6 Finding coordinates of points with the distance between two points given

In the figure, the distance between A and B is 6 units, and that between A and C is 7 units.



(a) Find the value of x.

(b) Find the value of y.

Solution

(a) $AB = 6$ units

$2 - x = 6$

$x = \underline{\underline{-4}}$

(b) $AC = 7$ units

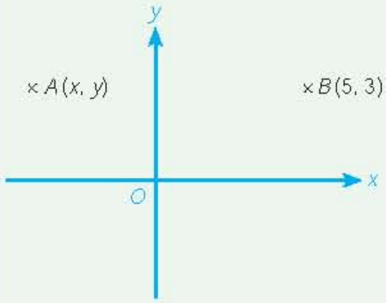
$4 - y = 7$

$y = \underline{\underline{-3}}$

Classwork 9.6

Solution

In the figure, AB is parallel to the x-axis, and the distance between A and B is 9 units.



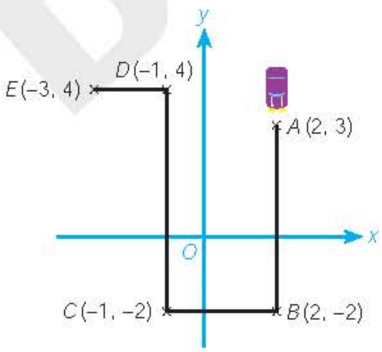
(a) Find the value of y.

(b) Find the value of x.

Additional Example 9.7

Example 9.7 Finding the total length of line segments

In the figure, a model car starts to move from A to E along line segments AB, BC, CD and DE. Find the total distance travelled by the model car.



Solution

$AB = [3 - (-2)]$ units
$= 5$ units

$BC = [2 - (-1)]$ units
$= 3$ units

$CD = [4 - (-2)]$ units
$= 6$ units

$DE = [(-1) - (-3)]$ units
$= 2$ units

$\therefore$ The total distance travelled by the model car

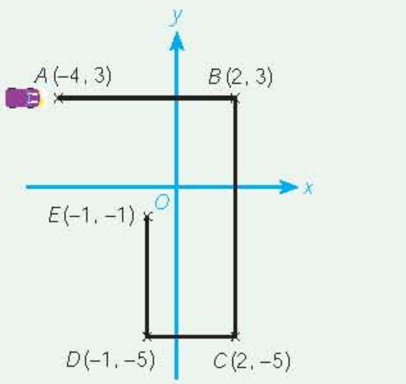
$= (5 + 3 + 6 + 2)$ units

$= \underline{\underline{16 \text{ units}}}$

Classwork 9.7

Solution

In the figure, a model car starts to move from A to E along line segments AB, BC, CD and DE. Find the total distance travelled by the model car.



Skills Upgrading Corner 9.2

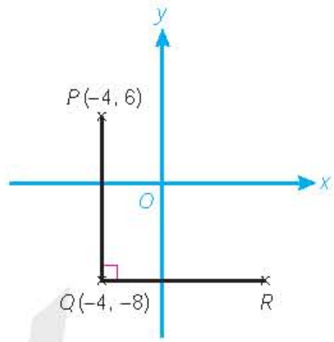
Solution

The figure shows the line segments PQ and QR, where $PQ \perp QR$ and $PQ = QR$.

(a) Find the length of PQ.

(b) Find the coordinates of R.

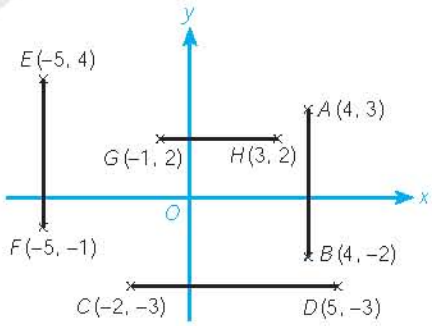
(c) If point S is added in the figure such that $PQRS$ forms a square, find the coordinates of S and the perimeter of square $PQRS$.



Exercise 9B

Level 1

1. In the figure, find the length of each line segment.



Find the distance between each pair of points below. **(2 – 4)**

2. **(a)** $A(2, 4)$, $B(6, 4)$ **(b)** $A(3, -2)$, $B(7, -2)$

(c) $A(3, 0)$, $B(-4, 0)$ **(d)** $A(-5, 1)$, $B(3, 1)$

(e) $A(-3, 4)$, $B(-8, 4)$ **(f)** $A(-4, -2)$, $B(-1, -2)$

3. **(a)** $P(5, 4)$, $Q(5, 2)$ **(b)** $P(1, 1)$, $Q(1, -4)$

(c) $P(0, 2)$, $Q(0, -2)$ **(d)** $P(-2, -4)$, $Q(-2, 3)$

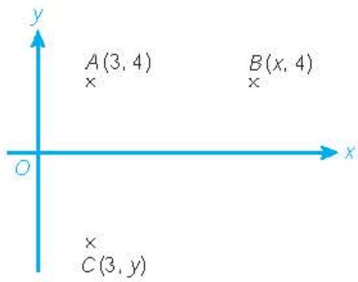
(e) $P(1, -1)$, $Q(1, -5)$ **(f)** $P(-4, -2)$, $Q(-4, -6)$

4. **(a)** $X(-5, 3.5)$, $Y(-5, -2.5)$ **(b)** $X(-4.3, 2)$, $Y(8, 2)$

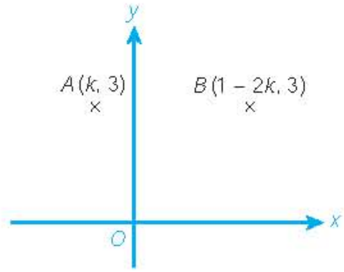
(c) $X(\frac{2}{5}, 6)$, $Y(\frac{2}{5}, -1)$ **(d)** $X(-\frac{1}{2}, \frac{1}{4})$, $Y(\frac{1}{2}, \frac{1}{4})$

Level 2

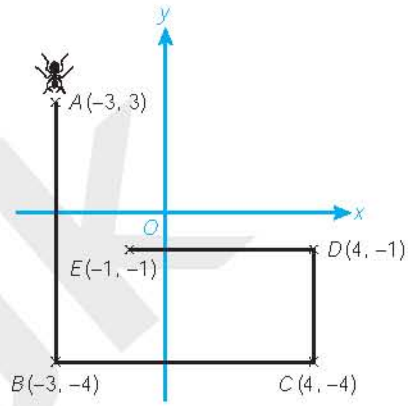
5. In the figure, the distances between points A and B and points A and C are both 9 units. Find the values of x and y.



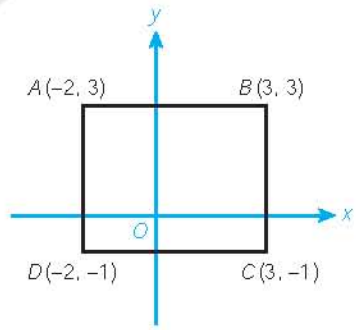
6. In the figure, the distance between $A(k, 3)$ and $B(1-2k, 3)$ is 4 units. Find the value of k.



7. In the figure, an ant starts at A and moves to E along line segments AB, BC, CD and DE. Find the total distance travelled by the ant.



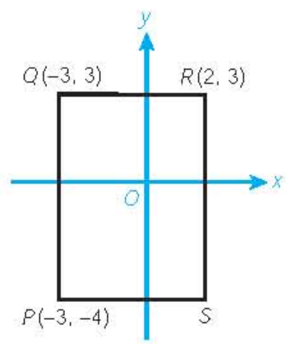
8. In the figure, $ABCD$ is a rectangle. Find its perimeter.



9. In the figure, $PQRS$ is a rectangle.

(a) Find the perimeter of rectangle $PQRS$.

(b) Find the coordinates of S.



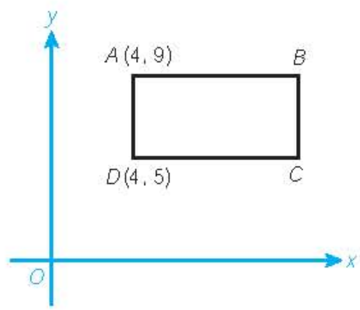
10. In the figure, $ABCD$ is a rectangle where $AB = 2AD$.

(a) Find the lengths of AD and AB.

(b) Find the perimeter of rectangle $ABCD$.

(c) Find the coordinates of B and C.

(d) If E is a point on DC such that $DE = EC$, find the coordinates of E.

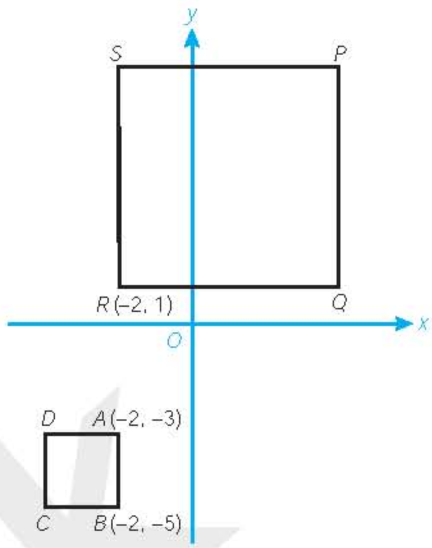


11. In the figure, $ABCD$ and $PQRS$ are squares. $RS = 3AB$, and RS is parallel to the y-axis.

(a) Find the lengths of AB and RS.

(b) Find the coordinates of C and D.

(c) Find the coordinates of P, Q and S.

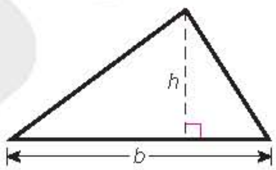


9.4 Areas of Polygons

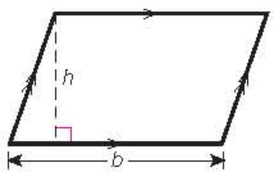
In primary schools, we have learned how to find the areas of some polygons.

e.g.

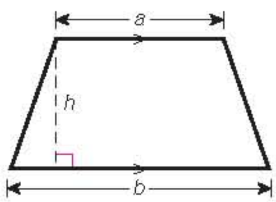
Area of a triangle $= \dfrac{1}{2}bh$



Area of a parallelogram $= bh$



Area of a trapezium $= \dfrac{(a+b)h}{2}$

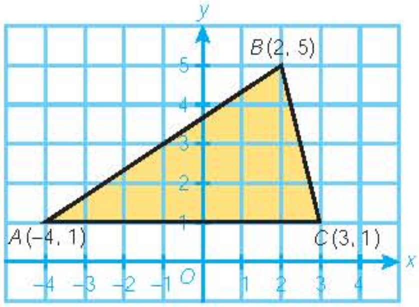


In this section, we will use these formulae to find the area of a polygon on a rectangular coordinate plane.

Additional Example 9.8

Example 9.8 Finding the area of a triangle using the formula directly

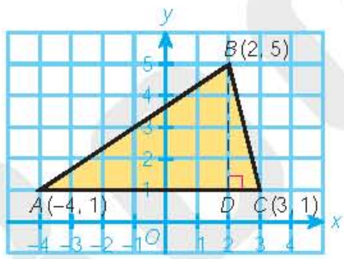
Find the area of ΔABC in the figure.



Solution

[Analysis: Draw a straight line BD, where D is a point on AC such that $BD \perp AC$.]

The coordinates of D are (2, 1).



$AC = [3 - (-4)]$ units

$= 7$ units

$BD = (5 - 1)$ units

$= 4$ units

$\therefore$ Area of ΔABC

$= \dfrac{1}{2} \times AC \times BD$

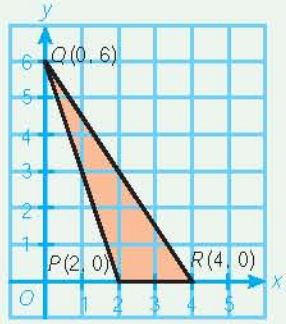
$= \dfrac{1}{2} \times 7 \times 4$ square units

$= \underline{\underline{14 \text{ square units}}}$

Solution

Classwork 9.8

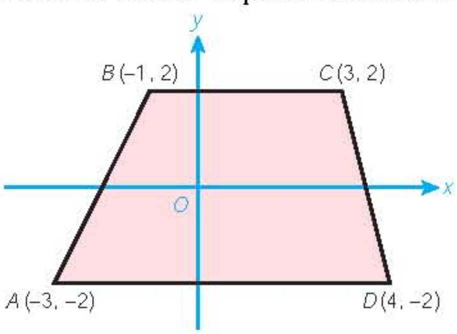
Find the area of ΔPQR in the figure.



Additional Example 9.9

Example 9.9 Finding the area of a trapezium using the formula directly

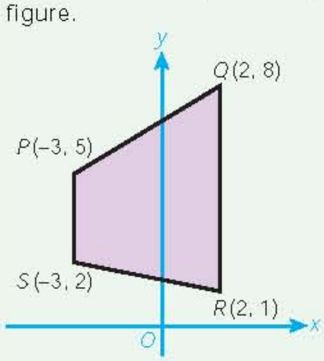
Find the area of trapezium $ABCD$ in the figure.



Solution

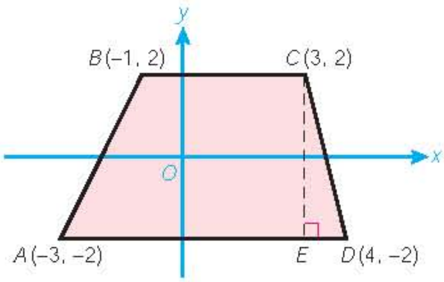
Classwork 9.9

Find the area of trapezium $PQRS$ in the figure.



Solution

[Analysis: Since AD and BC are parallel to the x-axis, AD // BC. Draw a straight line CE, where E is a point on AD such that $CE \perp AD$.]



The coordinates of E are $(3, -2)$.

$BC = [3 - (-1)]$ units
$\quad = 4$ units

$AD = [4 - (-3)]$ units
$\quad = 7$ units

$CE = [2 - (-2)]$ units
$\quad = 4$ units

$\therefore$ Area of trapezium $ABCD$

$$= \frac{1}{2} \times (BC + AD) \times CE$$

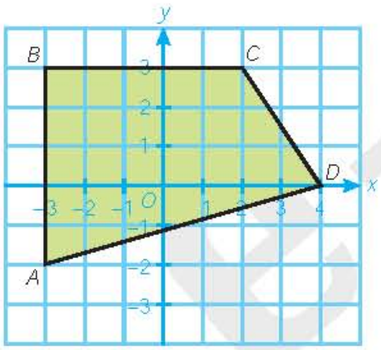
$$= \frac{1}{2} \times (4 + 7) \times 4 \text{ square units}$$

$$= \underline{\underline{22 \text{ square units}}}$$

Additional Example 9.10

Example 9.10 Finding the area of a quadrilateral by dissection

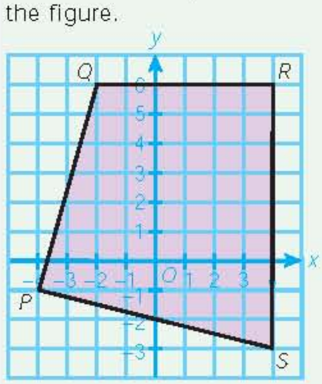
Find the area of quadrilateral $ABCD$ in the figure.



Classwork 9.10

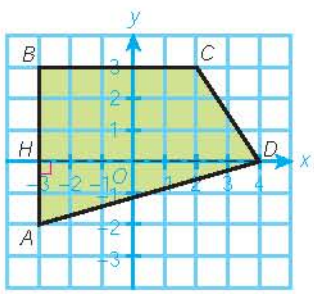
Solution

Find the area of quadrilateral $PQRS$ in the figure.



Solution

[Analysis: Divide $ABCD$ into trapezium $BCDH$ and ΔAHD.]



Area of trapezium $BCDH$

$$= \frac{1}{2} \times (BC + HD) \times BH$$

$$= \frac{1}{2} \times (5 + 7) \times 3 \text{ square units}$$

$$= 18 \text{ square units}$$

Area of ΔAHD

$= \frac{1}{2} \times HD \times HA$

$= \frac{1}{2} \times 7 \times 2$ square units

$= 7$ square units

$\therefore$ Area of quadrilateral $ABCD$

$=$ Area of trapezium $BCDH +$ Area of ΔAHD

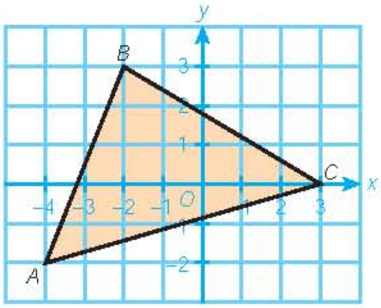
$= (18 + 7)$ square units

$= \underline{\underline{25 \text{ square units}}}$

Additional Example 9.11

Example 9.11 Finding the area of a triangle by the method of subtraction

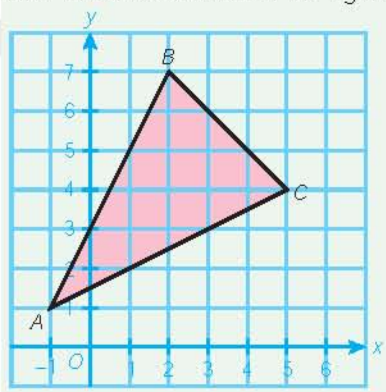
Find the area of ΔABC in the figure.



Classwork 9.11

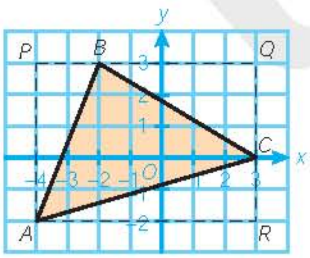
Solution

Find the area of ΔABC in the figure.



Solution

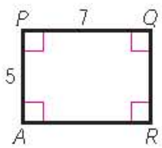
[Analysis: Draw a rectangle $APQR$ around ΔABC. Find the area of $APQR$ and then subtract the areas of the unshaded regions within $APQR$.]



Area of rectangle $APQR$

$= 7 \times 5$ square units

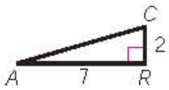
$= 35$ square units



Area of ΔACR

$= \frac{1}{2} \times 7 \times 2$ square units

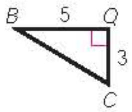
$= 7$ square units



Area of ΔBQC

$= \frac{1}{2} \times 3 \times 5$ square units

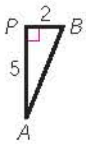
$= 7.5$ square units



Area of ΔAPB

$= \frac{1}{2} \times 5 \times 2$ square units

$= 5$ square units

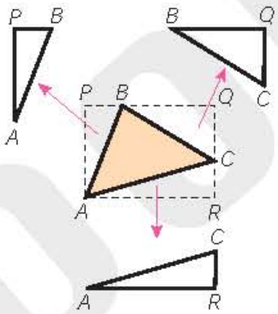


$\therefore$ Area of ΔABC

$=$ Area of rectangle $APQR$ – Area of ΔACR
– Area of ΔBQC – Area of ΔAPB

$= (35 - 7 - 7.5 - 5)$ square units

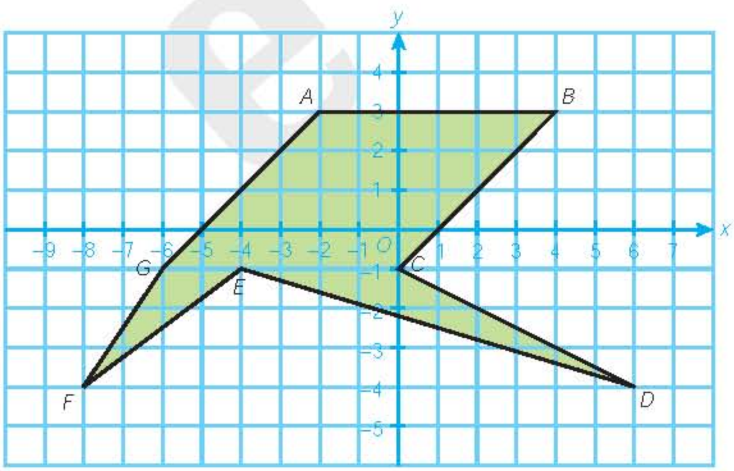
$= \underline{\underline{15.5 \text{ square units}}}$



Skills Upgrading Corner 9.3

Solution

Find the area of heptagon $ABCDEFG$ in the figure.



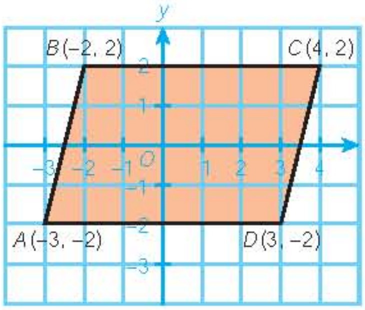
Exercise 9C

[Graph paper is provided in the Appendix.]

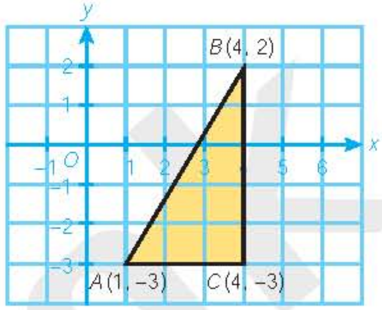
Level 1

1. Find the area of the shaded region in each of the following figures.

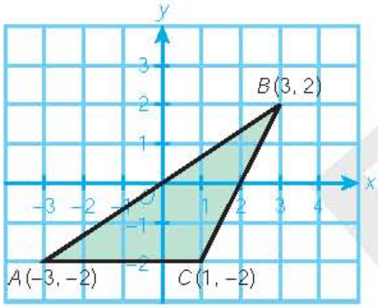
(a)



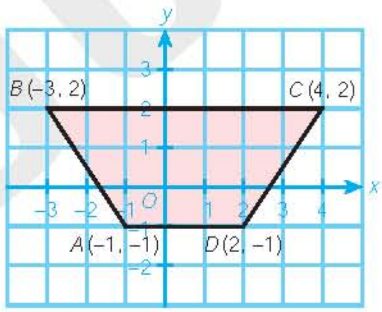
(b)



(c)



(d)

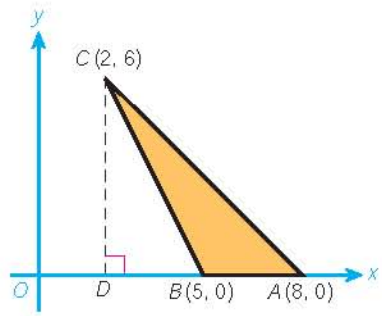


2. The figure shows ΔABC.

(a) Find the length of AB.

(b) Find the height CD of ΔABC.

(c) Find the area of ΔABC.

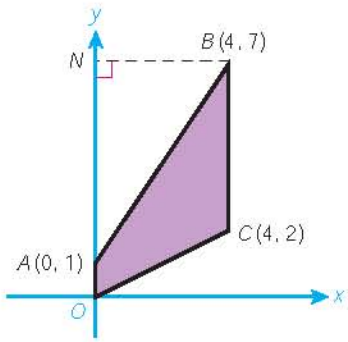


3. The figure shows trapezium $ABCO$.

(a) Find the lengths of OA and CB.

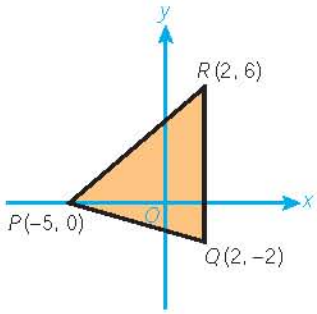
(b) If point N is on the y-axis such that BN is perpendicular to the y-axis, find the length of BN.

(c) Find the area of trapezium $ABCO$.

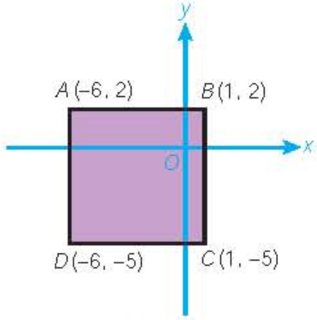


4. Find the area of the shaded region in each of the following figures.

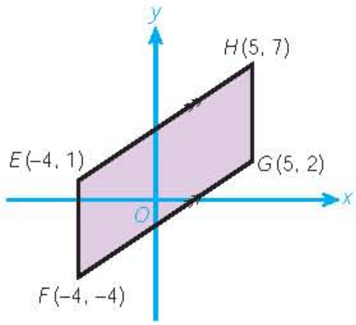
(a)



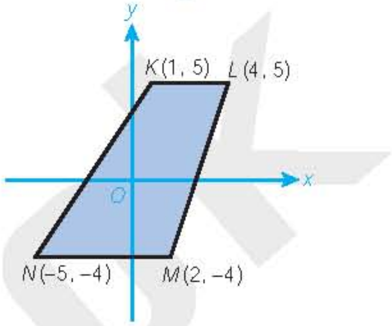
(b)



(c)



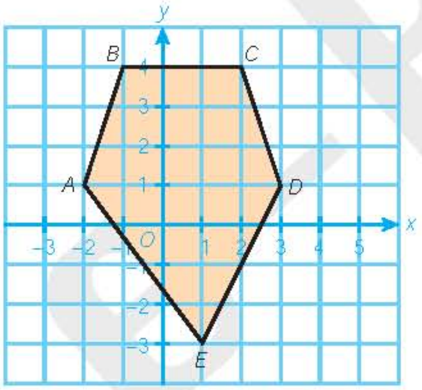
(d)



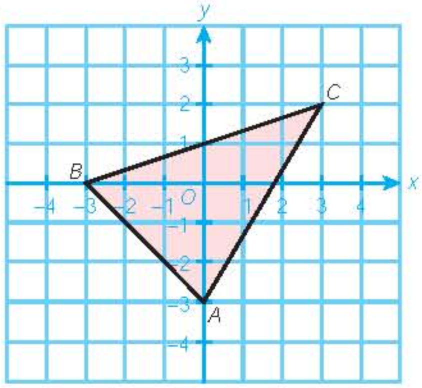
Level 2

Find the area of the shaded region in each of the following figures. (**5** – **6**)

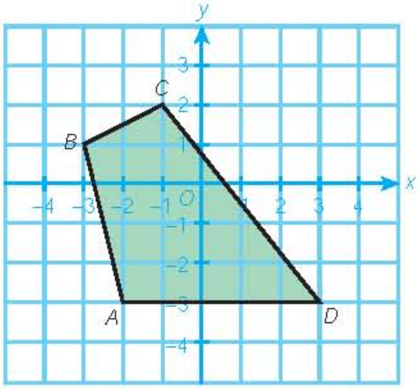
5. **(a)**



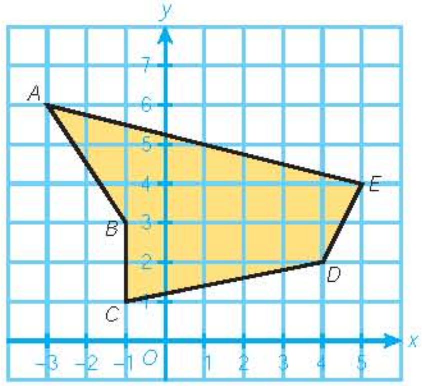
(b)



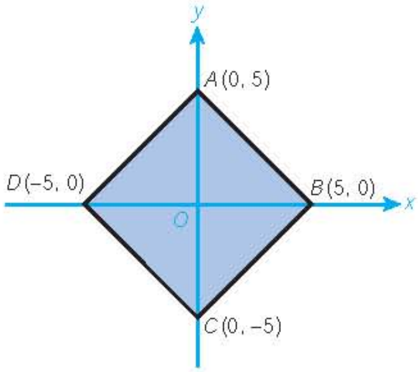
(c)



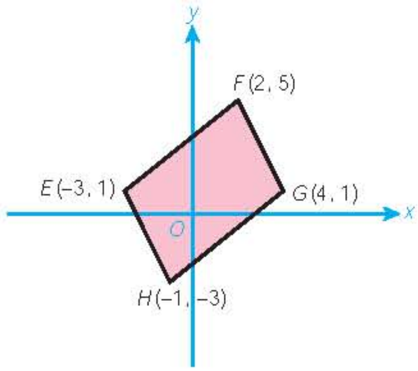
(d)



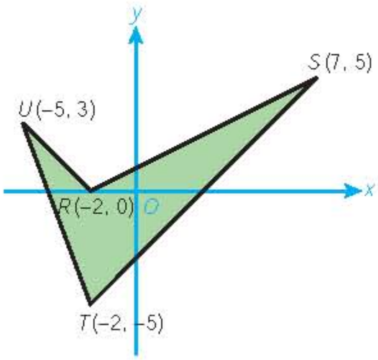
6. **(a)**



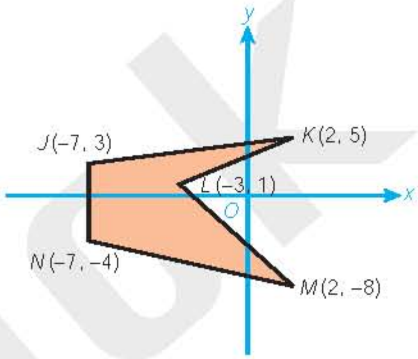
(b)



(c)



(d)



7. The following are the coordinates of the vertices of ΔPQR. Draw each triangle on a rectangular coordinate plane, and find its area.

(a) $P(1, 3)$, $Q(1, -2)$, $R(5, 0)$ **(b)** $P(0, 0)$, $Q(3, 4)$, $R(-3, 2)$

8. The following are the coordinates of the vertices of quadrilateral $ABCD$. Draw each quadrilateral on a rectangular coordinate plane, and find its area.

(a) $A(-2, -1)$, $B(-2, 2)$, $C(3, 5)$, $D(3, -2)$ **(b)** $A(0, 0)$, $B(1, 3)$, $C(4, 4)$, $D(5, 0)$

9. The following are the coordinates of the vertices of hexagon $EFGHIJ$. Draw each hexagon on a rectangular coordinate plane, and find its area.

(a) $E(2, 6)$, $F(4, 3)$, $G(2, -2)$, $H(-2, -2)$, $I(-4, 1)$, $J(-2, 6)$

(b) $E(-5, 3)$, $F(-3, 1)$, $G(2, 6)$, $H(2, -3)$, $I(-1, -1)$, $J(-5, -5)$

10. It is given that three vertices of a rectangle are $A(2, 1)$, $B(7, 1)$ and $C(2, 5)$.

(a) Mark the points A, B and C on a rectangular coordinate plane.

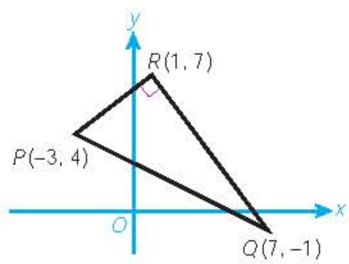
(b) Find the coordinates of the fourth vertex D.

(c) Find the area of rectangle $ABCD$.

11. The figure shows ΔPQR where $PR = 5$ units and $\angle PRQ = 90°$.

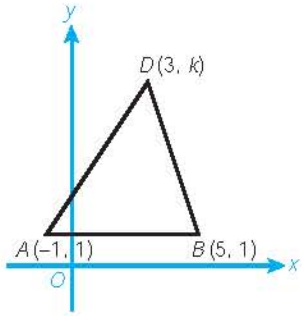
(a) Find the area of ΔPQR.

(b) Hence find the length of QR.

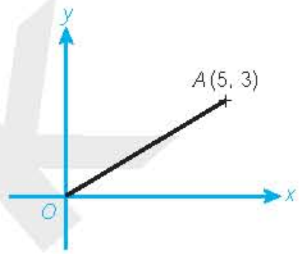


12. **(a)** In the figure, the area of ΔABD is 15 square units. Find the value of k.

(b) Point C is added in the figure such that $ABCD$ is a parallelogram with diagonal BD. Find the coordinates of C.



13. In the figure, OA is a side of ΔOAB. Find two possible sets of coordinates of B so that the area of ΔOAB is 6 square units. Explain your answer.

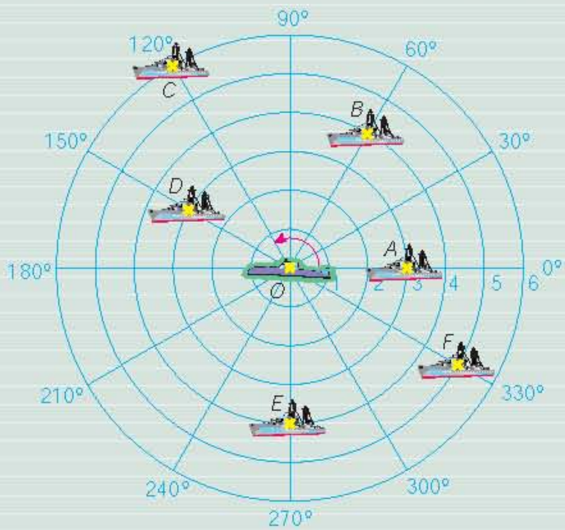


9.5 Polar Coordinate System

Class Activity 9.3

Aim: To recognize the polar coordinate system

In the figure, a submarine O is surrounded by six enemy warships A, B, C, D, E and F. Warship C is 6 units and the angle of direction is 120° from the submarine.



Fill in the table with the information on the location of the enemy warships. Show Ans.

Warship	A	B	C	D	E	F
Distance			6			
Angle of direction			120°			

Now I see ...

In addition to the rectangular coordinate system, there are other coordinate systems to determine the positions of points on a plane.

I.T. Tool

The **polar coordinate system** is a coordinate system which uses the distance and angle of a reference point to determine the positions of points.

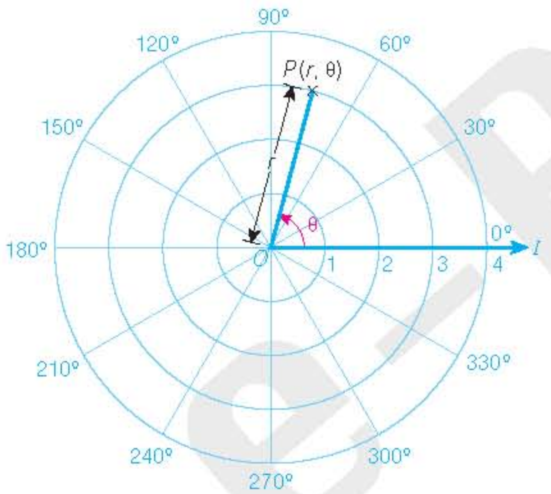


Figure 9.7

In Figure 9.7,

1. O is a fixed point called the **pole**.
2. OI is a fixed line called the **polar axis**.
3. The distance from O to P is called the **radius vector** r.
4. The angle between OP and OI is called the **vectorial angle** θ which measures from OI to OP anti-clockwise.
5. The position of P is represented by an ordered pair (r, θ), called the **polar coordinates** of P.

θ (read as theta) is usually used to denote an unknown angle.

For the ordered pair (r, θ), the radius vector r should be written first and then the vectorial angle θ. Therefore the polar coordinates of O are $(0, 0°)$.

polar coordinate system 極坐標系統
radius vector 極徑
pole 極
vectorial angle 極角
polar axis 極軸
polar coordinates 極坐標

Figure 9.8 shows the positions of some points on a **polar coordinate plane**.

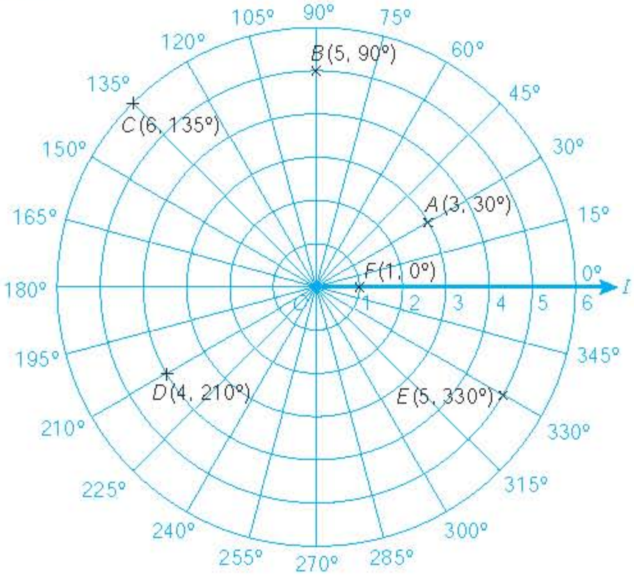
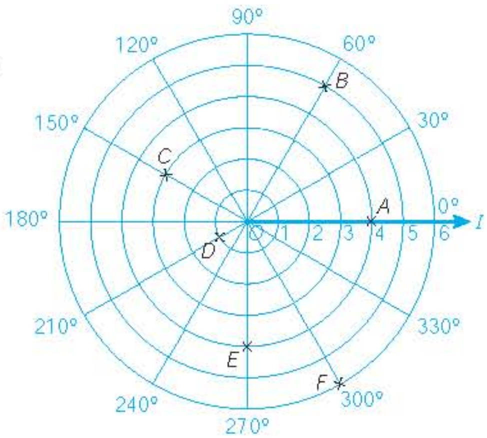


Figure 9.8

Extension 9.2

Solution

The figure shows a polar coordinate plane.



(a) Write down the polar coordinates of points A to F in the figure.

A: ________________

B: ________________

C: ________________

D: ________________

E: ________________

F: ________________

(b) Mark two points $G(4, 120°)$ and $H(2, 330°)$ on the polar coordinate plane.

(c) Find $\angle BOG$ and $\angle COH$.

(d) Find the distance between C and H.

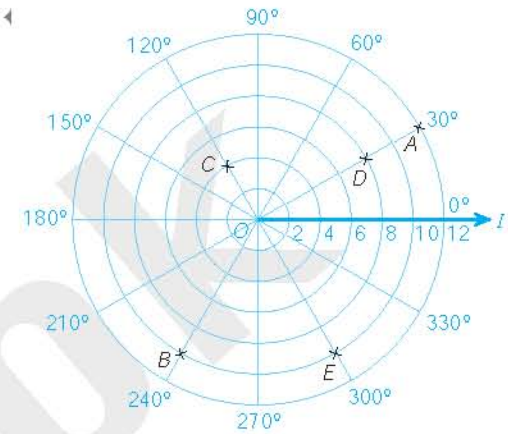
polar coordinate plane 極坐標平面

Exercise 9D

[A polar coordinate plane is provided in the Appendix.]

Level 1

1. The figure shows a polar coordinate plane. Write down the polar coordinates of points A to E.



2. **(a)** Mark four points $A(3, 30°)$, $B(3, 120°)$, $C(3, 240°)$ and $D(3, 330°)$ on a polar coordinate plane.
   (b) What is common among these points?

3. **(a)** Mark four points $P(1, 60°)$, $Q(2, 60°)$, $R(3, 60°)$ and $S(4, 60°)$ on a polar coordinate plane.
   (b) What is common among these points?

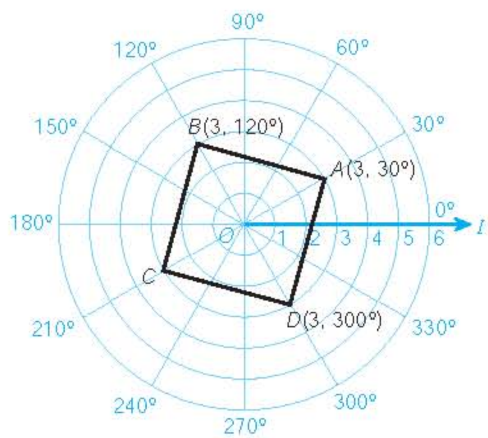
4. **(a)** Mark four points $W(1, 120°)$, $X(3, 120°)$, $Y(2, 300°)$ and $Z(4, 300°)$ on a polar coordinate plane.
   (b) Join the points according to the order W, X, Y, Z and W. What is the figure obtained?

Level 2

5. **(a)** Mark three points $O(0, 0°)$, $A(6, 30°)$ and $B(6, 150°)$ on a polar coordinate plane.
   (b) What kind of triangle is OAB?

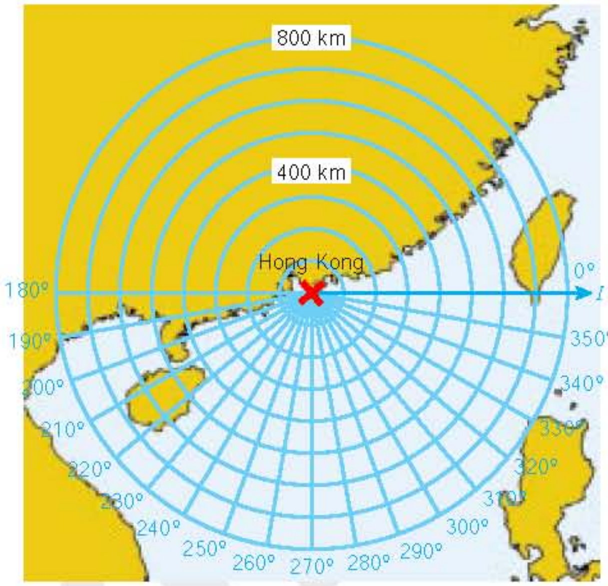
6. The polar coordinates of A, B and C are $(2, 30°)$, $(6, 30°)$ and $(3, 210°)$ respectively.
   (a) **(i)** Find the distance between A and B.
   (ii) Find the distance between A and C.
   (b) Which point, B or C, is farther away from A?

7. **(a)** In the figure, $A(3, 30°)$, $B(3, 120°)$, C and $D(3, 300°)$ on the polar coordinate plane form a square. Find the polar coordinates of C.
   (b) Find $\angle AOB$.
   (c) Find the area of square $ABCD$.



8. The table records the locations of the centre of typhoon Dragon Ball at different times.

Date (Time)	Polar coordinates
1/7 (8:00 am)	(800, 320°)
1/7 (8:00 pm)	(600, 310°)
2/7 (8:00 am)	(400, 300°)
2/7 (8:00 pm)	(300, 280°)
3/7 (8:00 am)	(300, 240°)
3/7 (8:00 pm)	(400, 210°)
4/7 (8:00 am)	(600, 200°)
4/7 (8:00 pm)	(700, 190°)
5/7 (8:00 am)	(800, 180°)



(a) On the polar coordinate plane with Hong Kong as the pole, mark the locations of Dragon Ball at different times.
[A copy of the figure is provided in the Appendix.]

(b) When Dragon Ball came within 800 km of Hong Kong, the observatory hoisted warning signal No.1. What was the time then?

(c) According to the given information, when was Dragon Ball closest to Hong Kong?

(d) When did Dragon Ball start to land? How far was it from Hong Kong at that moment?

9.6 Transformations on a Rectangular Coordinate Plane

In Chapter 8, we have learned some basic transformations of plane figures. In this section, we try to observe the changes of the coordinates of points and plane figures on a rectangular coordinate plane after transformations.

A Translation

In Figure 9.9, each point on the solid-lined triangle moves in the same direction and distance to obtain the dotted-lined triangle. This kind of transformation is called translation.

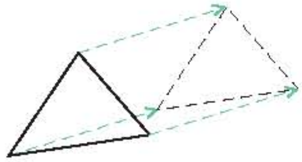


Figure 9.9

In Figure 9.10, line segment AB is translated 4 units to the right and 1 unit upwards to $A'B'$.

Under the translation,

$A(2,\ 1)$ is transformed to $A'(2+4,\ 1+1)$, i.e. $A'(6,\ 2)$;

$B(3,\ 2)$ is transformed to $B'(3+4,\ 2+1)$, i.e. $B'(7,\ 3)$.

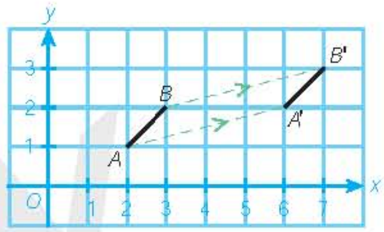
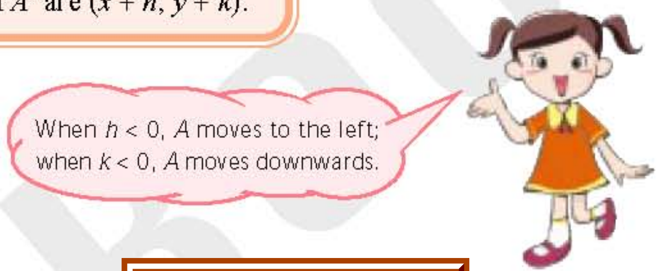


Figure 9.10

Therefore, on a rectangular coordinate plane,

> If the coordinates of A are (x, y), and A is translated h units to the right and k units upwards to the image A', then the coordinates of A' are $(x + h, y + k)$.



Additional Example 9.12

Example 9.12 Changes in coordinates after translation

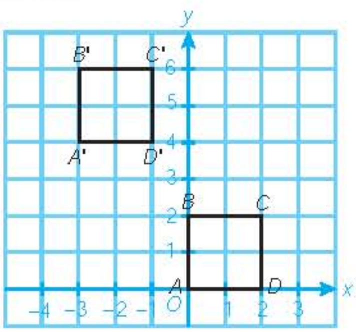
The vertices of square $ABCD$ are $A(0,\ 0)$, $B(0,\ 2)$, $C(2,\ 2)$ and $D(2,\ 0)$. Square $ABCD$ is translated 3 units to the left and 4 units upwards to obtain the image $A'B'C'D'$.

(a) Draw $ABCD$ and $A'B'C'D'$ on the same rectangular coordinate plane, and find the coordinates of the vertices of image $A'B'C'D'$.

(b) Compare the positions, shapes and sizes of $ABCD$ and $A'B'C'D'$.

Solution

(a)



Coordinates of $A' = \underline{\underline{(-3,\ 4)}}$

Coordinates of $B' = \underline{\underline{(-3,\ 6)}}$

Coordinates of $C' = \underline{\underline{(-1,\ 6)}}$

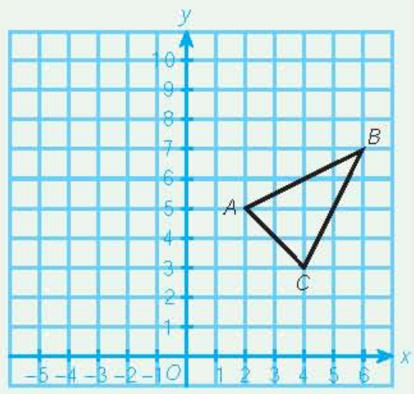
Coordinates of $D' = \underline{\underline{(-1,\ 4)}}$

(b) The positions of $ABCD$ and $A'B'C'D'$ are different but their shapes and sizes are the same.

Classwork 9.12

Solution

In the figure, ΔABC is translated 4 units to the left and 2 units upwards to obtain the image $\Delta A'B'C'$. Draw $\Delta A'B'C'$ in the figure and find the coordinates of the vertices of $\Delta A'B'C'$.



B Reflection

In Figure 9.11, straight line AB is like a mirror and the solid-lined figures are reflected along AB to obtain the dotted-lined figures. This kind of transformation is called reflection.

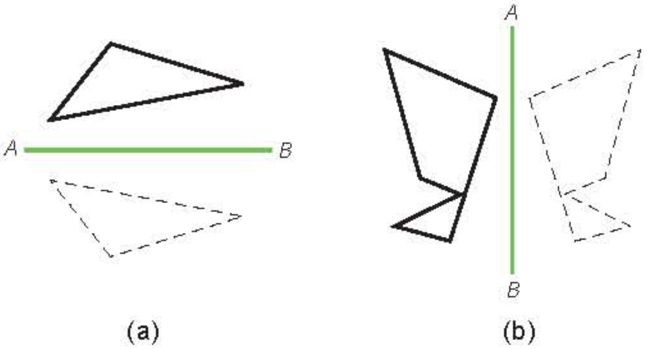


Figure 9.11

I.T. Tool

I. Reflection along the x-axis or y-axis

In Figure 9.12, $A'B'$ is the image of reflecting AB along the x-axis, under which

$A(2, 1)$ is transformed to $A'(2, -1)$;

$B(3, 2)$ is transformed to $B'(3, -2)$.

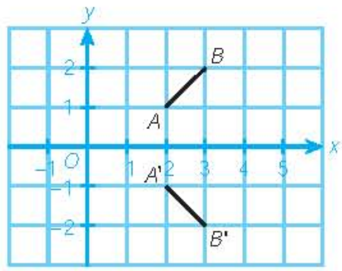


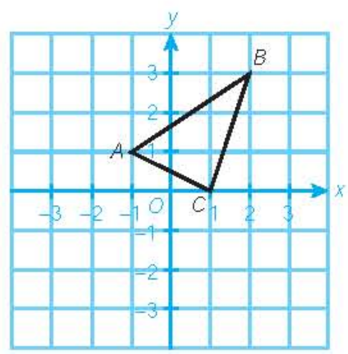
Figure 9.12

Therefore, on a rectangular coordinate plane,

> If the coordinates of A are (x, y), and A is reflected along the x-axis to the image A', then the coordinates of A' are $(x, -y)$.

Extension 9.3 Solution

In the figure, ΔABC is reflected along the x-axis to obtain the image $\Delta A'B'C'$. Draw $\Delta A'B'C'$ in the figure, and find the coordinates of the vertices of $\Delta A'B'C'$.



In Figure 9.13, $A'B'$ is the image of reflecting AB along the y-axis, under which

$A(2, 1)$ is transformed to $A'(-2, 1)$;

$B(3, 2)$ is transformed to $B'(-3, 2)$.

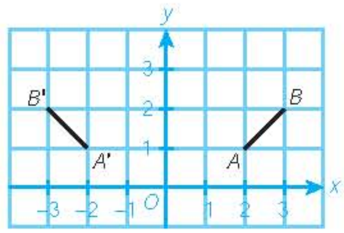


Figure 9.13

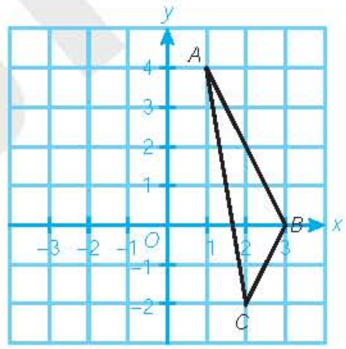
Therefore, on a rectangular coordinate plane,

> If the coordinates of A are (x, y) and A is reflected along the y-axis to the image A', then the coordinates of A' are $(-x, y)$.

Extension 9.4

Solution

In the figure, ΔABC is reflected along the y-axis to obtain the image $\Delta A'B'C'$. Draw $\Delta A'B'C'$ in the figure, and find the coordinates of the vertices of $\Delta A'B'C'$.



II. Reflection along a straight line parallel to a coordinate axis

In Figure 9.14, straight line L is parallel to the x-axis and is 3 units above the x-axis. $A'B'$ is the image of reflecting AB along straight line L, under which

$A(2, 1)$ is transformed to $A'(2, 5)$;

$B(3, 2)$ is transformed to $B'(3, 4)$.

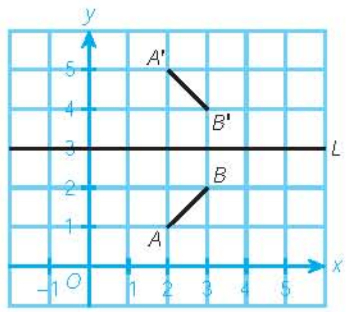


Figure 9.14

In Figure 9.15, straight line L is parallel to the y-axis and is 1 unit to the right of the y-axis. $A'B'$ is the image of reflecting AB along straight line L, under which

$A(2, 1)$ is transformed to $A'(0, 1)$;

$B(3, 2)$ is transformed to $B'(-1, 2)$.

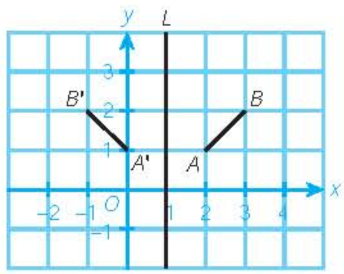
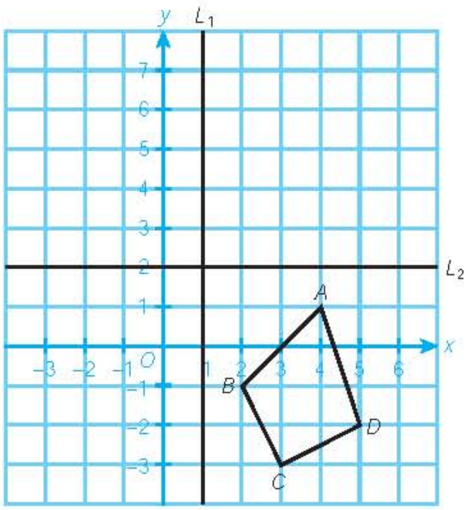


Figure 9.15

Additional Example 9.13

Example 9.13 Changes in coordinates after reflection

(a) In the figure, quadrilateral $ABCD$ is reflected along straight line L_1 to obtain the image $A_1B_1C_1D_1$. Draw $ABCD$ and $A_1B_1C_1D_1$ on the same rectangular coordinate plane.

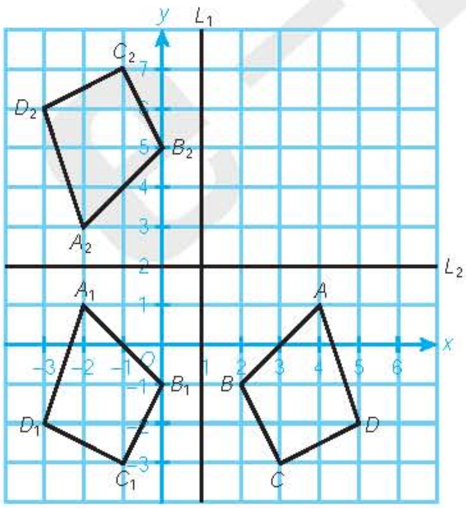


(b) If $A_2B_2C_2D_2$ is the image of reflecting $A_1B_1C_1D_1$ obtained in **(a)** along straight line L_2, draw $A_2B_2C_2D_2$ on the rectangular coordinate plane in **(a)**.

(c) Find the coordinates of D_2.

Solution

(a), (b)

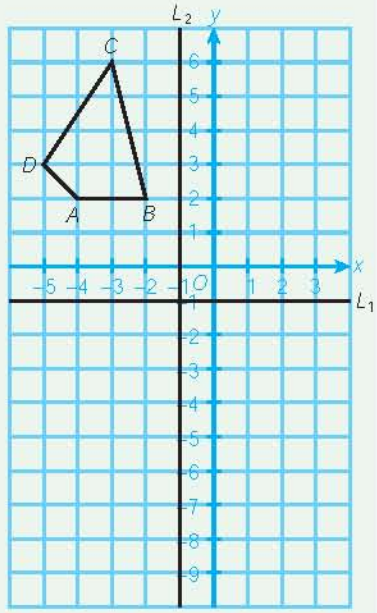


(c) Coordinates of $D_2 = \underline{\underline{(-3,\ 6)}}$

Classwork 9.13

Solution

(a) In the figure, quadrilateral $ABCD$ is reflected along straight line L_1 to obtain the image $A_1B_1C_1D_1$. Draw $A_1B_1C_1D_1$ in the figure.



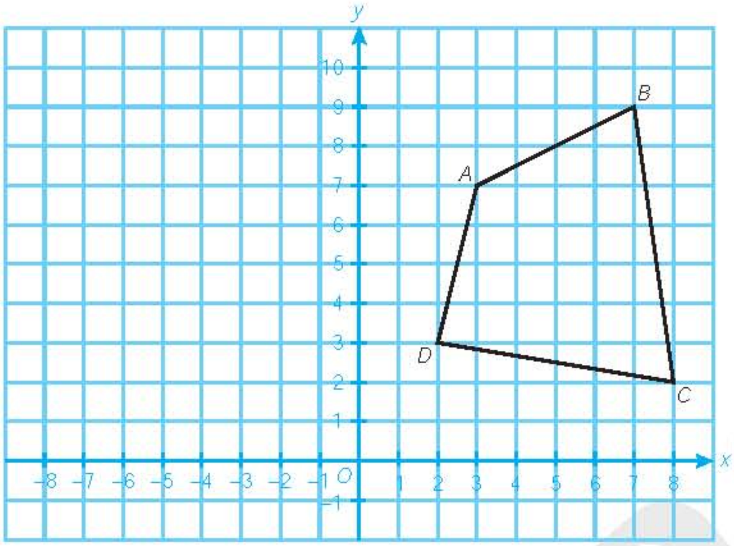
(b) If $A_2B_2C_2D_2$ is the image of reflecting $A_1B_1C_1D_1$ obtained in (a) along straight line L_2, draw $A_2B_2C_2D_2$ on the rectangular coordinate plane in (a).

(c) Find the coordinates of D_2.

Skills Upgrading Corner 9.4

Solution

(a) In the figure, quadrilateral $ABCD$ is reflected along the y-axis to obtain the image $A_1B_1C_1D_1$. Draw $A_1B_1C_1D_1$ in the figure.



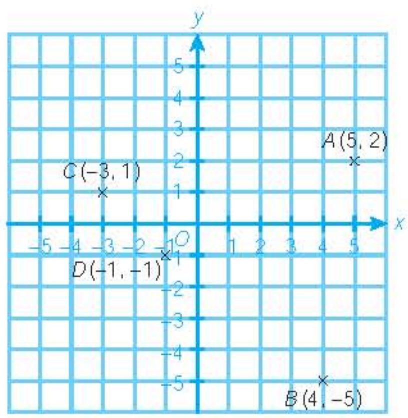
(b) If $A_1B_1C_1D_1$ obtained in **(a)** is translated 5 units to the right and 3 units downwards to obtain the image $A_2B_2C_2D_2$, draw $A_2B_2C_2D_2$ in the figure.

Exercise 9E

[Graph paper is provided in the Appendix.]

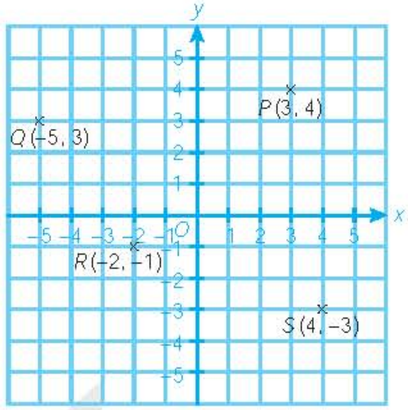
Level 1

1. The figure shows the positions of points A to D. Based on the following table, find the coordinates of the image of each point after translation.



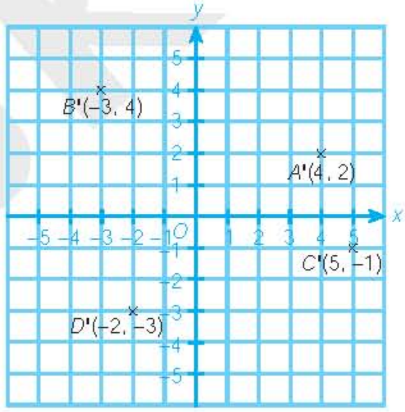
	Point	Translation	Image
(a)	A	Translate 2 units to the left and 3 units downwards	A'
(b)	B	Translate 2 units to the left and 3 units upwards	B'
(c)	C	Translate 4 units to the right and 5 units downwards	C'
(d)	D	Translate 4 units to the right and 5 units upwards	D'

2. The figure shows the positions of points P to S. Based on the following table, find the coordinates of the image of each point after reflection.



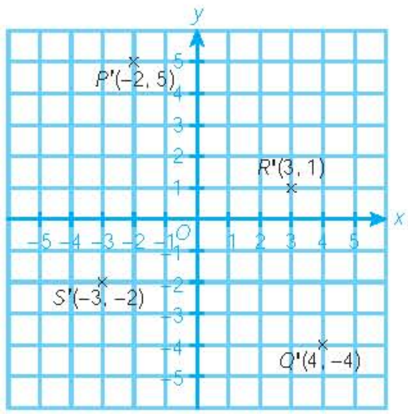
	Point	Reflection	Image
(a)	P	Reflect along the x-axis	P'
(b)	Q	Reflect along the x-axis	Q'
(c)	R	Reflect along the y-axis	R'
(d)	S	Reflect along the y-axis	S'

3. The figure shows the positions of images A' to D' after translating points A to D respectively. Based on the following table, find the coordinates of each point before translation.



	Point	Translation	Image
(a)	A	Translate 1 unit to the left and 2 units downwards	A'
(b)	B	Translate 2 units to the left and 3 units upwards	B'
(c)	C	Translate 4 units to the right and 2 units downwards	C'
(d)	D	Translate 3 units to the right and 1 unit upwards	D'

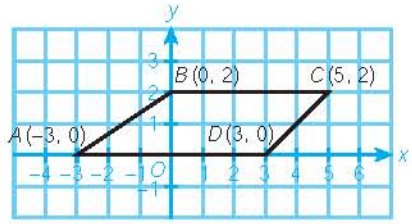
4. The figure shows the positions of images P' to S' after reflecting points P to S respectively. Based on the following table, find the coordinates of each point before reflection.



	Point	Reflection	Image
(a)	P	Reflect along the x-axis	P'
(b)	Q	Reflect along the x-axis	Q'
(c)	R	Reflect along the y-axis	R'
(d)	S	Reflect along the y-axis	S'

Level 2

5. Quadrilateral $ABCD$ in the figure is transformed in each of the following ways to obtain the image $A'B'C'D'$. For each transformation, draw $ABCD$ and $A'B'C'D'$ on the same rectangular coordinate plane, and find the coordinates of the vertices of $A'B'C'D'$.

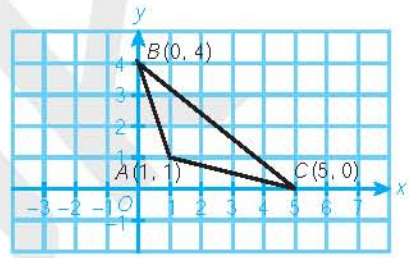


(a) Translate 1 unit to the right and 2 units upwards.

(b) Translate 3 units to the left and 1 unit upwards.

(c) Translate 2 units to the right and 1 unit downwards.

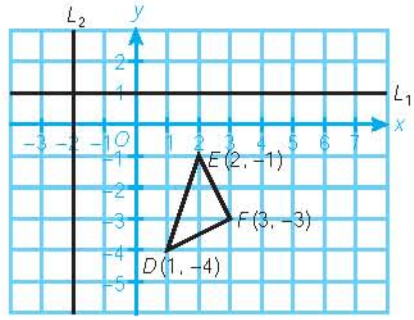
6. ΔABC in the figure is transformed in each of the following ways to obtain the image $\Delta A'B'C'$. For each transformation, draw ΔABC and $\Delta A'B'C'$ on the same rectangular coordinate plane, and find the coordinates of the vertices of $\Delta A'B'C'$.



(a) Reflect along the x-axis.

(b) Reflect along the y-axis.

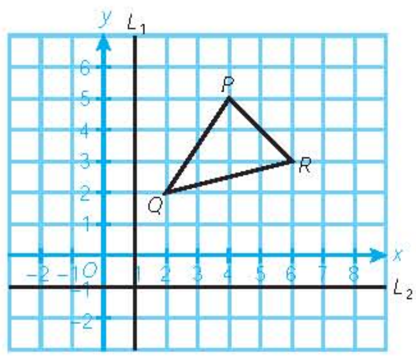
7. ΔDEF in the figure is transformed in each of the following ways to obtain the image $\Delta D'E'F'$. For each transformation, draw ΔDEF and $\Delta D'E'F'$ on the same rectangular coordinate plane, and find the coordinates of the vertices of $\Delta D'E'F'$.



(a) Reflect along straight line L_1.

(b) Reflect along straight line L_2.

8. **(a)** In the figure, ΔPQR is reflected along straight line L_1 to obtain the image $\Delta P_1Q_1R_1$. Draw ΔPQR and $\Delta P_1Q_1R_1$ on the same rectangular coordinate plane.



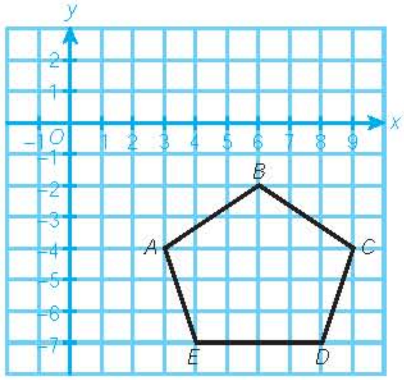
(b) If $\Delta P_2Q_2R_2$ is the image of reflecting $\Delta P_1Q_1R_1$ obtained in **(a)** along straight line L_2, draw $\Delta P_2Q_2R_2$ on the rectangular coordinate plane in **(a)**.

(c) Find the coordinates of R_2.

9. **(a)** In the figure, pentagon $ABCDE$ is reflected along the x-axis to obtain the image $A_1B_1C_1D_1E_1$. Draw $ABCDE$ and $A_1B_1C_1D_1E_1$ on the same rectangular coordinate plane.

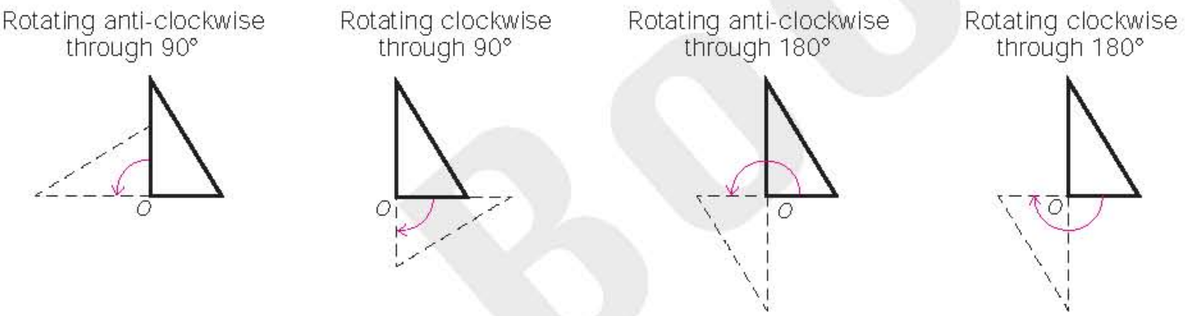
(b) If $A_1B_1C_1D_1E_1$ obtained in **(a)** is translated 6 units to the left and 3 units downwards to obtain the image $A_2B_2C_2D_2E_2$, draw $A_2B_2C_2D_2E_2$ on the rectangular coordinate plane in **(a)**.

(c) If pentagon $ABCDE$ is translated 6 units to the left and 3 units downwards, and then reflected along the x-axis, does the image obtained overlap $A_2B_2C_2D_2E_2$? Explain briefly.



C Rotation

In Figure 9.16, the solid-lined figures are rotated about the centre of rotation O to obtain the dotted-lined figures.



(a) (b) (c) (d)

Figure 9.16

I.T. Tool

I. Rotation anti-clockwise about the origin through 90°

In Figure 9.17, $\Delta A'B'C'$ is the image of rotating ΔABC anti-clockwise about the origin through 90°, under which

$A(2, 1)$ is transformed to $A'(-1, 2)$;
$B(4, 1)$ is transformed to $B'(-1, 4)$;
$C(4, 5)$ is transformed to $C'(-5, 4)$.

Therefore, on a rectangular coordinate plane,

> If the coordinates of A are (x, y), and A is rotated anti-clockwise about the origin through 90° to obtain the image A', then the coordinates of A' are $(-y, x)$.

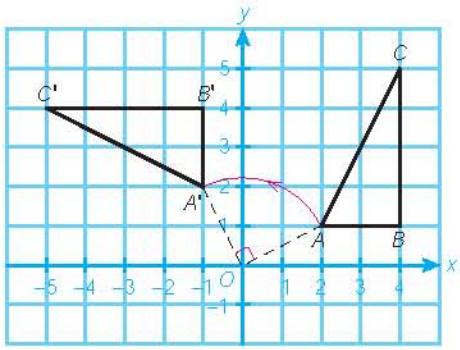


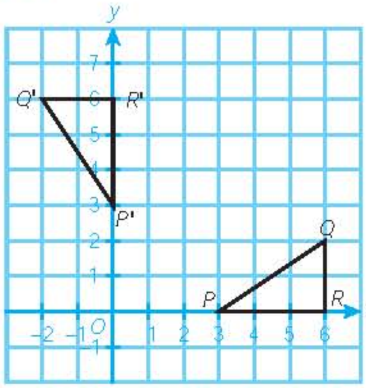
Figure 9.17

Additional Example 9.14

Example 9.14 Changes in coordinates after rotating anti-clockwise about the origin through 90°

The vertices of ΔPQR are $P(3,\ 0)$, $Q(6,\ 2)$ and $R(6,\ 0)$. ΔPQR is rotated anti-clockwise about the origin through 90° to obtain the image $\Delta P'Q'R'$. Draw ΔPQR and $\Delta P'Q'R'$ on the same rectangular coordinate plane, and find the coordinates of the vertices of $\Delta P'Q'R'$.

Solution



Coordinates of $P' = \underline{\underline{(0,\ 3)}}$

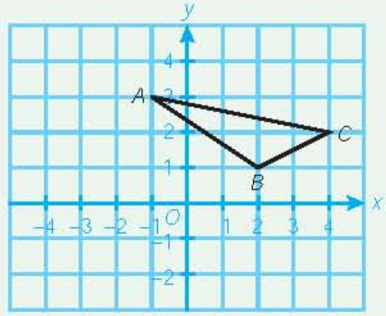
Coordinates of $Q' = \underline{\underline{(-2,\ 6)}}$

Coordinates of $R' = \underline{\underline{(0,\ 6)}}$

Classwork 9.14

Solution

In the figure, ΔABC is rotated anti-clockwise about the origin through 90° to obtain the image $\Delta A'B'C'$. Draw $\Delta A'B'C'$ in the figure, and find the coordinates of the vertices of $\Delta A'B'C'$.



II. Rotation clockwise about the origin through 90°

In Figure 9.18, $\Delta A'B'C'$ is the image of rotating ΔABC clockwise about the origin through 90°, under which

$A(2,\ 1)$ is transformed to $A'(1,\ -2)$;

$B(4,\ 1)$ is transformed to $B'(1,\ -4)$;

$C(4,\ 5)$ is transformed to $C'(5,\ -4)$.

Therefore, on a rectangular coordinate plane,

> If the coordinates of A are $(x,\ y)$, and A is rotated clockwise about the origin through 90° to obtain the image A', then the coordinates of A' are $(y,\ -x)$.

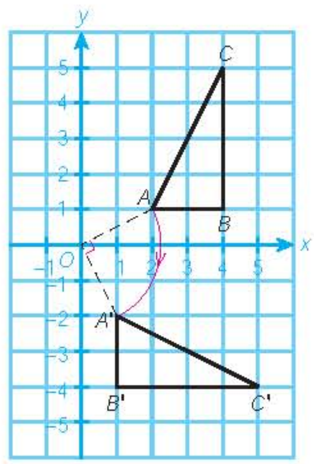


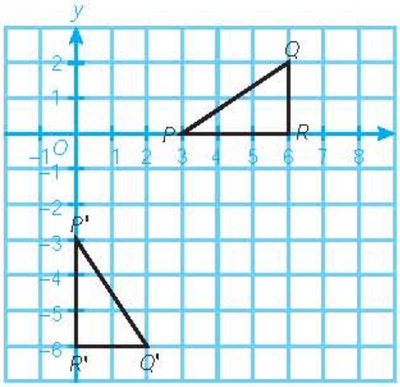
Figure 9.18

Additional Example 9.15

Example 9.15 Changes in coordinates after rotating clockwise about the origin through 90°

The vertices of ΔPQR are $P(3, 0)$, $Q(6, 2)$ and $R(6, 0)$. ΔPQR is rotated clockwise about the origin through 90° to obtain the image $\Delta P'Q'R'$. Draw ΔPQR and $\Delta P'Q'R'$ on the same rectangular coordinate plane, and find the coordinates of the vertices of $\Delta P'Q'R'$.

Solution



Coordinates of $P' = \underline{\underline{(0, -3)}}$

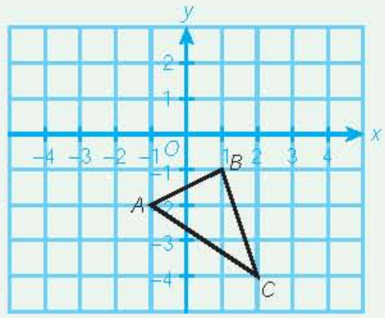
Coordinates of $Q' = \underline{\underline{(2, -6)}}$

Coordinates of $R' = \underline{\underline{(0, -6)}}$

Classwork 9.15

Solution

In the figure, ΔABC is rotated clockwise about the origin through 90° to obtain the image $\Delta A'B'C'$. Draw $\Delta A'B'C'$ in the figure, and find the coordinates of the vertices of $\Delta A'B'C'$.



III. Rotation about the origin through 180°

In Figure 9.19, quadrilateral $A'B'C'D'$ is the image of rotating quadrilateral $ABCD$ about the origin through 180°, under which

$A(1, 2)$ is transformed to $A'(-1, -2)$;

$B(4, 1)$ is transformed to $B'(-4, -1)$;

$C(2, 3)$ is transformed to $C'(-2, -3)$;

$D(2, 4)$ is transformed to $D'(-2, -4)$.

Therefore, on a rectangular coordinate plane,

> If the coordinates of A are (x, y), and A is rotated about the origin through 180° to obtain the image A', then the coordinates of A' are $(-x, -y)$.

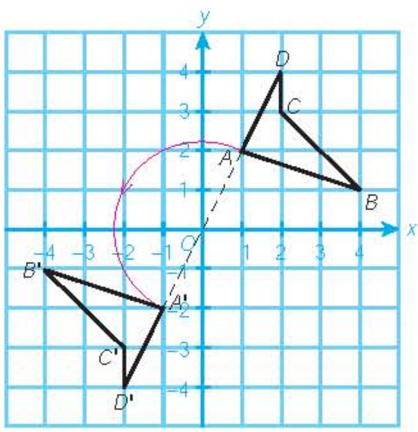
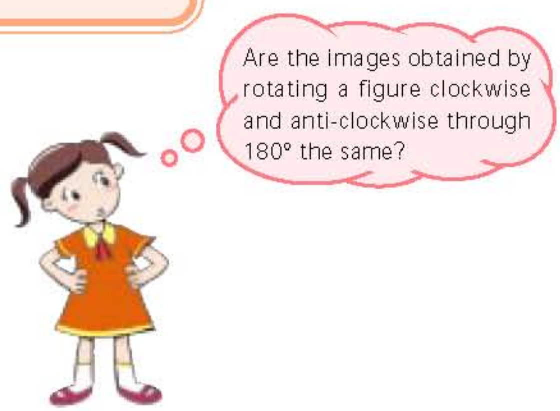


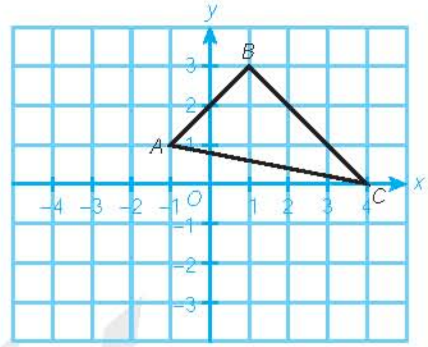
Figure 9.19



Extension 9.5

Solution

In the figure, ΔABC is rotated about the origin through $180°$ to obtain the image $\Delta A'B'C'$. Draw $\Delta A'B'C'$ in the figure, and find the coordinates of the vertices of $\Delta A'B'C'$.



Additional Example 9.16

Example 9.16 Changes in coordinates after rotation and reflection

The vertices of ΔOAB are $O(0, 0)$, $A(4, 2)$ and $B(4, 0)$. ΔOAB is rotated clockwise about the origin through $90°$ to obtain the image ΔOA_1B_1. ΔOA_1B_1 is then reflected along the x-axis to obtain the image ΔOA_2B_2.

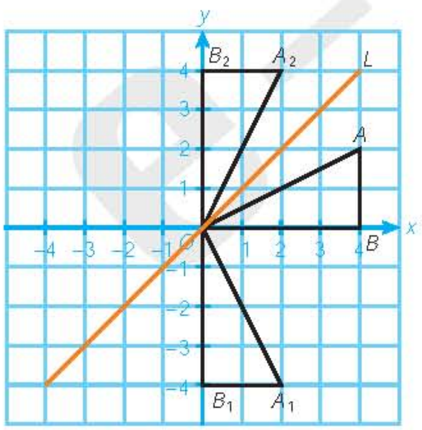
(a) Draw ΔOAB, ΔOA_1B_1 and ΔOA_2B_2 on the same rectangular coordinate plane.

(b) Find the coordinates of A_1, B_1, A_2 and B_2.

(c) Do ΔOAB and ΔOA_2B_2 have the property of reflectional symmetry? If they do, draw the axis of symmetry.

Solution

(a), (c)



(b) Coordinates of $A_1 = \underline{\underline{(2,\ -4)}}$

Coordinates of $B_1 = \underline{\underline{(0,\ -4)}}$

Coordinates of $A_2 = \underline{\underline{(2,\ 4)}}$

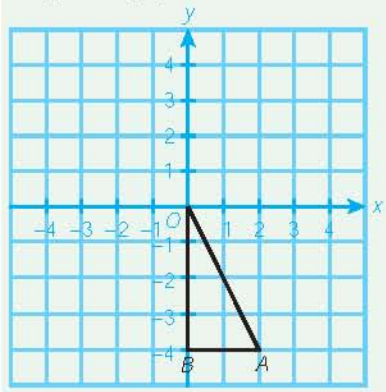
Coordinates of $B_2 = \underline{\underline{(0,\ 4)}}$

(c) ΔOAB and ΔOA_2B_2 have the property of reflectional symmetry. The axis of symmetry is straight line L.

Classwork 9.16

Solution

In the figure, ΔOAB is reflected along the y-axis to obtain the image ΔOA_1B_1. ΔOA_1B_1 is rotated clockwise about the origin through $90°$ to obtain the image ΔOA_2B_2.



(a) Draw ΔOA_1B_1 and ΔOA_2B_2 in the figure.

(b) Find the coordinates of A_1, B_1, A_2 and B_2.

(c) Do ΔOAB and ΔOA_2B_2 have the property of reflectional symmetry? If they do, draw the axis of symmetry.

Skills Upgrading Corner 9.5

Solution

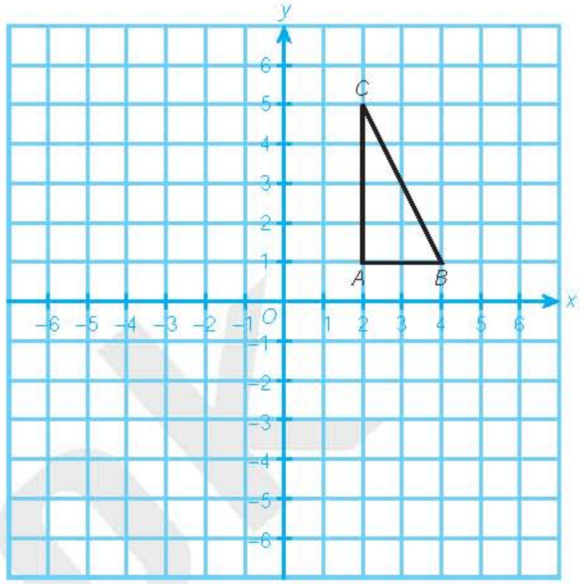
In the figure, ΔABC is rotated about the origin through 180°, and then translated 6 units to the right and 6 units upwards to obtain the image $\Delta A'B'C'$.

(a) Draw $\Delta A'B'C'$ in the figure.

(b) Does quadrilateral $AB'A'B$ have the following properties?

(i) Reflectional symmetry

(ii) Rotational symmetry

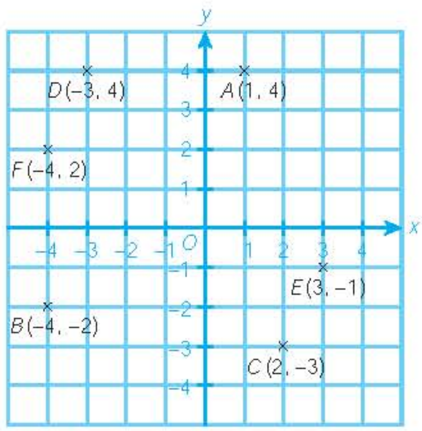


Exercise 9F

[Graph paper is provided in the Appendix.]

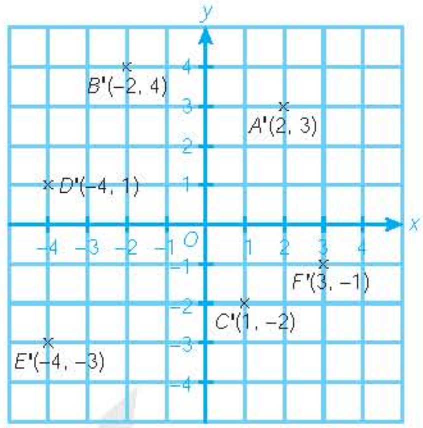
Level 1

1. The figure shows the positions of points A to F. Based on the following table, find the coordinates of the image of each point after rotation.



	Point	Rotation (about the origin)	Image
(a)	A	Rotate anti-clockwise through 90°	A'
(b)	B	Rotate anti-clockwise through 90°	B'
(c)	C	Rotate clockwise through 90°	C'
(d)	D	Rotate clockwise through 90°	D'
(e)	E	Rotate through 180°	E'
(f)	F	Rotate through 180°	F'

2. The figure shows the positions of images A' to F' after rotating points A to F respectively. Based on the following table, find the coordinates of each point before rotation.



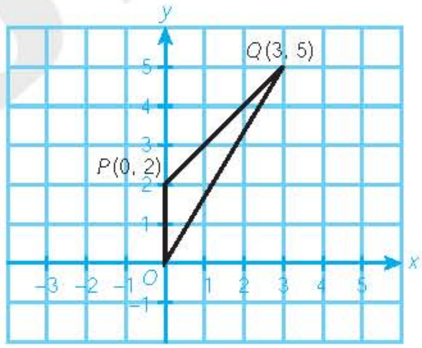
	Point	Rotation (about the origin)	Image
(a)	A	Rotate clockwise through 90°	A'
(b)	B	Rotate clockwise through 90°	B'
(c)	C	Rotate anti-clockwise through 90°	C'
(d)	D	Rotate anti-clockwise through 90°	D'
(e)	E	Rotate through 180°	E'
(f)	F	Rotate through 180°	F'

3. ΔOPQ in the figure is transformed in each of the following ways to obtain the image $\Delta OP'Q'$. For each transformation, draw ΔOPQ and $\Delta OP'Q'$ on the same rectangular coordinate plane, and find the coordinates of P' and Q'.

(a) Rotate anti-clockwise about the origin through 90°.

(b) Rotate clockwise about the origin through 90°.

(c) Rotate about the origin through 180°.



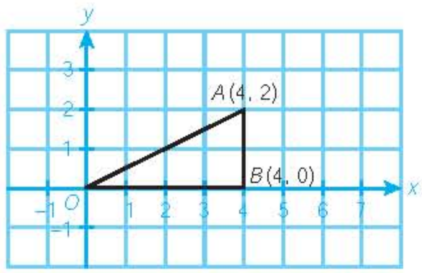
Level 2

4. In the figure, ΔOAB is rotated anti-clockwise about the origin through 90° to obtain the image ΔOA_1B_1. Then ΔOA_1B_1 is reflected along the x-axis to obtain the image ΔOA_2B_2.

(a) Draw ΔOAB, ΔOA_1B_1 and ΔOA_2B_2 on the same rectangular coordinate plane.

(b) Find the coordinates of A_1, B_1, A_2 and B_2.

(c) Do ΔOAB and ΔOA_2B_2 have the property of reflectional symmetry? If they do, draw the axis of symmetry.



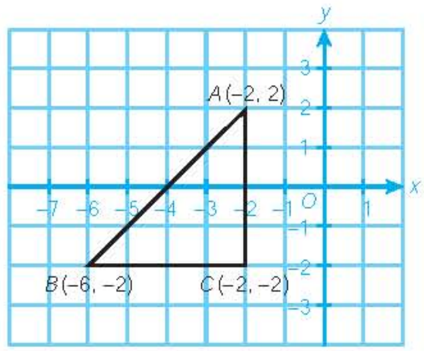
5. In the figure, ΔABC is translated 4 units to the right, and then rotated about the origin through 180° to obtain the image $\Delta A'B'C'$.

(a) Draw ΔABC and $\Delta A'B'C'$ on the same rectangular coordinate plane.

(b) Does quadrilateral $AB'A'B$ have the following properties?

(i) Reflectional symmetry

(ii) Rotational symmetry



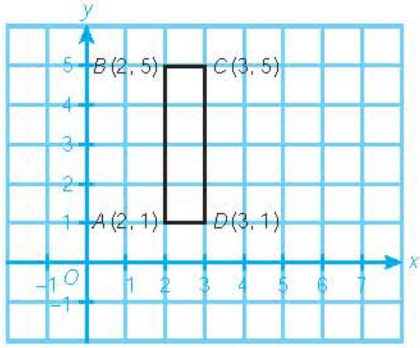
6. In the figure, rectangle $ABCD$ is reflected along the x-axis to obtain the image $A_1B_1C_1D_1$. Then $A_1B_1C_1D_1$ is reflected along the y-axis to obtain the image $A_2B_2C_2D_2$.

(a) Draw $ABCD$, $A_1B_1C_1D_1$ and $A_2B_2C_2D_2$ on the same rectangular coordinate plane.

(b) **(i)** Find the coordinates of the vertices of image $A_1B_1C_1D_1$.

(ii) Find the coordinates of the vertices of image $A_2B_2C_2D_2$.

(c) If $ABCD$ changes to $A_2B_2C_2D_2$ after a single transformation, describe the transformation.



Chapter Summary

Chapter Summary

A. Term Introduced

[This is a quiz to check your understanding of some special terms in this chapter. Match items in column A to column B appropriately.]

Column A			Column B
1. Ordered pair	•	•	**(a)** A system using two perpendicular lines to locate the position of a point.
2. Rectangular coordinate system	•	•	**(b)** Written in the form (x, y), and can be used to describe the positions of points.
3. Quadrant	•	•	**(c)** A system using distance and angle to locate the position of a point.
4. Polar coordinate system	•	•	**(d)** One of the four regions obtained by dividing a rectangular coordinate plane with the coordinate axes.

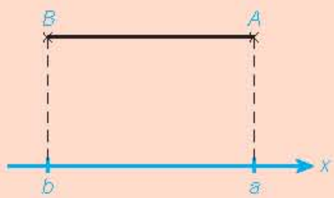
B. Fact to Remember

1. If $x \neq y$, and (x, y) is an ordered pair, then $(x,\ y) \neq (y,\ x)$.

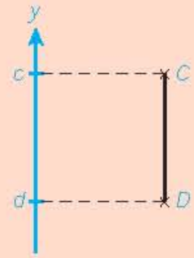
2. Distances

(a) Two points on a horizontal line

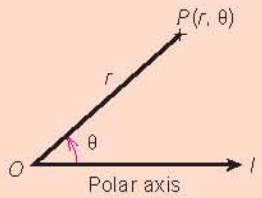
If A and B are two points on a horizontal line, A is on the right of B, and the x-coordinates of A and B are a and b respectively, then $AB = a - b$.



(b) Two points on a vertical line
If C and D are two points on a vertical line, C is above D, and the y-coordinates of C and D are c and d respectively, then $CD = c - d$.



3. In the polar coordinate system, the vectorial angle θ is measured anti-clockwise.



4. When $A(x, y)$ is under the following transformations, the coordinates of its image are as follows:

(a) Translation: Translate h units to the right and k units upwards, the image is $A'(x + h, y + k)$.

(b) Reflection : Reflect along the x-axis, the image is $A'(x, -y)$;
reflect along the y-axis, the image is $A'(-x, y)$.

(c) Rotation: Rotate anti-clockwise about the origin through 90°, the image is $A'(-y, x)$;
rotate clockwise about the origin through 90°, the image is $A'(y, -x)$;
rotate about the origin through 180°, the image is $A'(-x, -y)$.

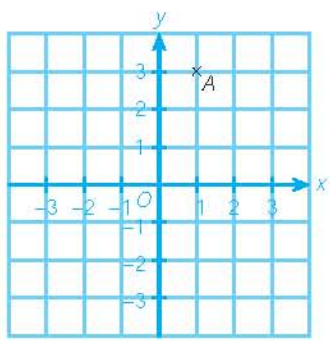
Check Yourself

[This is a quiz to remind you of the basic concepts you have learned in this chapter. Each question tests a concept under the section listed on the right. Failure in any part of a question indicates a need to do a revision on the section listed.]

Section

1. (a) (i) In the figure, the coordinates of A are (________, ________). 9.2

(ii) Mark point $B(-3, -1)$ on the rectangular coordinate plane.



(b) Match items in column A to column B appropriately.

Column A			**Column B**
(3, 5) lies	•	•	on the x-axis
(−5, 4) lies	•	•	on the y-axis
(−3, 0) lies	•	•	in quadrant I
(3, −1) lies	•	•	in quadrant II
(−2, −3) lies	•	•	in quadrant III
(0, 4) lies	•	•	in quadrant IV

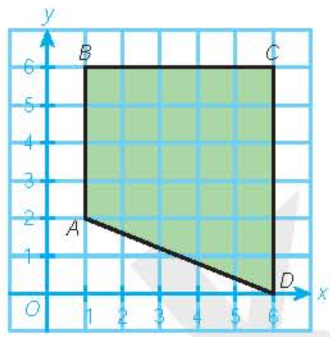
2. Four points $A(-3, -2)$, $B(2, -2)$, $C(4, 2)$ and $D(4, 6)$ are given. Find the distance between each pair of points below. 9.3

(a) A and B

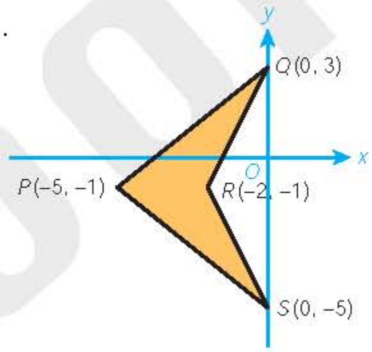
(b) C and D

3. **(a)** In the figure, the area of trapezium $ABCD$ is 9.4

__________ square units.



(b) Find the area of quadrilateral $PQRS$ in the figure.

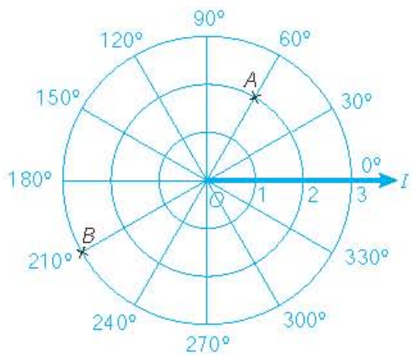


4. The figure shows a polar coordinate plane. 9.5

(a) The polar coordinates of A are

(__________, __________).

(b) Find $\angle AOB$.



5. **(a)** If the point $A(1, 3)$ is translated 3 units to the left, then the new coordinates of A are ____________________. 9.6A

(b) $E(0, -2)$ is translated 6 units to the right and 5 units downwards to obtain the image E'. Find the coordinates of E'.

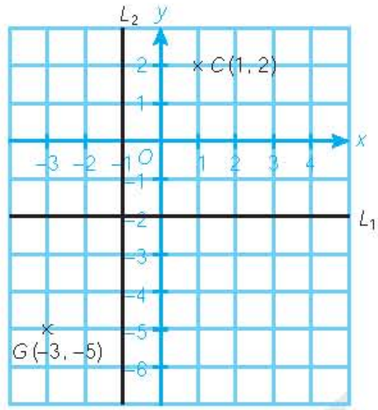
6. **(a)** If the point $B(3, -4)$ is reflected along the x-axis to B', then the coordinates of B' are ____________________. 9.6B

(b) $F(-5, 6)$ is reflected along the y-axis to obtain the image F'. Find the coordinates of F'.

7. The figure shows a rectangular coordinate plane. 9.6B

(a) If $C(1, 2)$ is reflected along straight line L_1 to C', then the coordinates of C' are ____________________.

(b) $G(-3, -5)$ is reflected along straight line L_2 to obtain the image G'. Find the coordinates of G'.



8. **(a)** If $D(-2, -1)$ is rotated clockwise about $(0, 0)$ through $90°$ to D', 9.6C

then the coordinates of D' are ____________________.

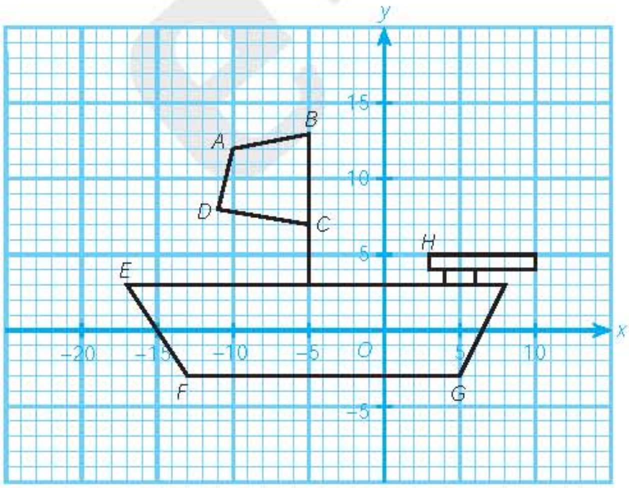
(b) $H(-7, 4)$ is rotated about the origin through $180°$ to obtain the image H'. Find the coordinates of H'.

Revision Exercise 9

[Graph paper and a polar coordinate plane are provided in the Appendix.]

Level 1

1. Write down the coordinates of points A to H in the figure.



2. **(a)** Mark two points $A(-3, 4)$ and $B(-2, 2)$ on a rectangular coordinate plane.

(b) Join AB.

(c) Produce line segment AB and write down the coordinates of the points where AB cuts the x-axis and the y-axis.

3. **(a)** Mark four points $A(2, 1)$, $B(3, 3)$, $C(-2, 3)$ and $D(-3, 1)$ on a rectangular coordinate plane.

(b) Join the points according to the order A, B, C, D and A.

(c) What kind of quadrilateral is $ABCD$?

(d) Write down the coordinates of the point of intersection of the diagonals of $ABCD$.

4. Find the distance between each pair of points below.

(a) $A(1, 2)$, $B(1, 6)$ **(b)** $P(-4, 3)$, $Q(7, 3)$

(c) $X(-2, 5)$, $Y(-2, -3)$ **(d)** $S(-5, -6)$, $T(4, -6)$

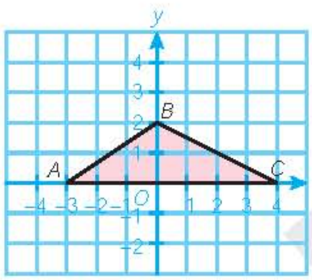
5. It is given that three vertices of rectangle $ABCD$ are $A(-8, 5)$, $B(-2, 5)$ and $C(-2, -3)$.

(a) Mark the points A, B and C on a rectangular coordinate plane.

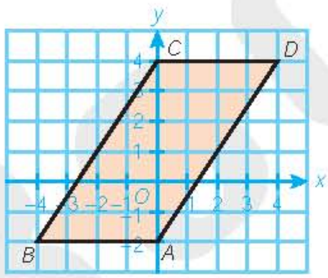
(b) Find the coordinates of the fourth vertex D.

6. Find the area of the shaded region in each of the following figures.

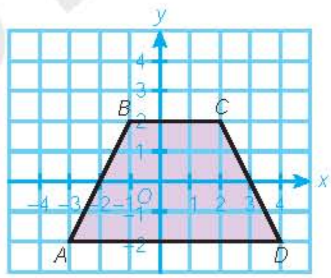
(a)



(b)



(c)



7. **(a)** Mark three points $A(4, 30°)$, $B(4, 150°)$ and $C(4, 270°)$ on a polar coordinate plane.

(b) What kind of triangle is ABC?

8. For the following points, find the coordinates of their images after transformation as follows.

	Coordinates before transformation	Transformation
(a)	(3, 9)	Translate 3 units to the left and 2 units upwards
(b)	(−9, 6)	Translate 5 units to the right and 4 units downwards
(c)	(5, −1)	Reflect along the x-axis
(d)	(−4, 2)	Reflect along the y-axis
(e)	(2, −1)	Rotate anti-clockwise about the origin through 90°
(f)	(−3, 6)	Rotate clockwise about the origin through 90°
(g)	(−1, −3)	Rotate about the origin through 180°

9. For the following images, find the coordinates of the points before transformation as follows.

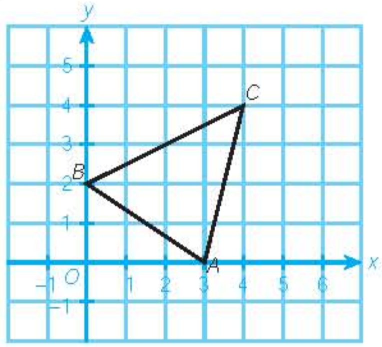
	Transformation	Coordinates after transformation
(a)	Translate 3 units to the left and 2 units upwards	$(-4, -9)$
(b)	Translate 5 units to the right and 4 units downwards	$(2, -5)$
(c)	Reflect along the x-axis	$(2, 6)$
(d)	Reflect along the y-axis	$(3, -8)$
(e)	Rotate anti-clockwise about the origin through 90°	$(-12, 10)$
(f)	Rotate clockwise about the origin through 90°	$(7, 5)$
(g)	Rotate about the origin through 180°	$(4, -8)$

10. ΔABC in the figure is transformed in each of the following ways to obtain the image $\Delta A'B'C'$. For each transformation, draw ΔABC and $\Delta A'B'C'$ on the same rectangular coordinate plane, and find the coordinates of the vertices of image $\Delta A'B'C'$.

(a) Translate 2 units to the right and 1 unit upwards.

(b) Translate 3 units to the left and 3 units upwards.

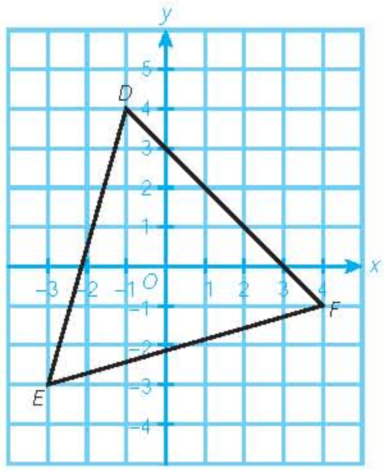
(c) Translate 4 units to the right and 2 units downwards.



11. ΔDEF in the figure is transformed in each of the following ways to obtain the image $\Delta D'E'F'$. For each transformation, draw ΔDEF and $\Delta D'E'F'$ on the same rectangular coordinate plane, and find the coordinates of the vertices of image $\Delta D'E'F'$.

(a) Reflect along the x-axis.

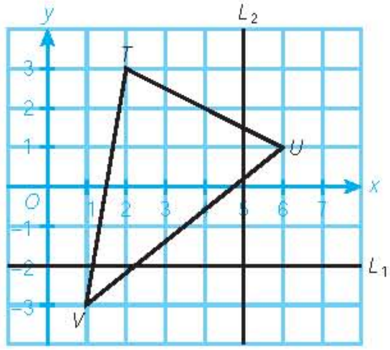
(b) Reflect along the y-axis.



12. ΔTUV in the figure is transformed in each of the following ways to obtain the image $\Delta T'U'V'$. For each transformation, draw ΔTUV and $\Delta T'U'V'$ on the same rectangular coordinate plane, and find the coordinates of the vertices of image $\Delta T'U'V'$.

(a) Reflect along straight line L_1.

(b) Reflect along straight line L_2.

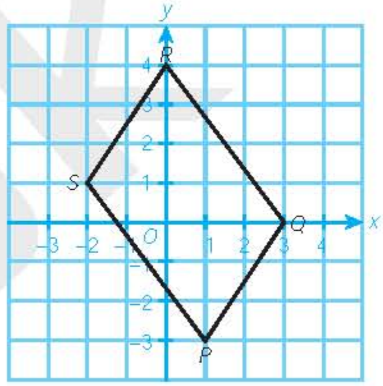


13. Quadrilateral $PQRS$ in the figure is transformed in each of the following ways to obtain the image $P'Q'R'S'$. For each transformation, draw $PQRS$ and $P'Q'R'S'$ on the same rectangular coordinate plane, and find the coordinates of the vertices of image $P'Q'R'S'$.

(a) Rotate anti-clockwise about the origin through 90°.

(b) Rotate clockwise about the origin through 90°.

(c) Rotate about the origin through 180°.



Level 2

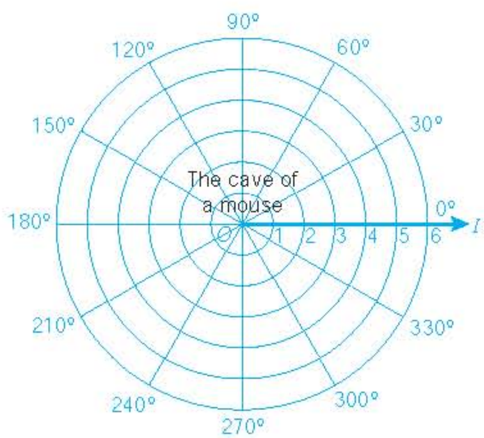
14. **(a)** Mark four points $A(3, 45°)$, $B(3, 135°)$, $C(3, 225°)$ and $D(3, 315°)$ on a polar coordinate plane.

(b) Join the points according to the order A, B, C, D and A. What kind of quadrilateral is $ABCD$?

(c) Find the area of quadrilateral $ABCD$.

15. In the figure, the cave of a mouse is located at the pole of a polar coordinate plane. A snake is moving from (5, 60°) to the cave. The following table shows the locations of the snake at the different times.

Time (min)	0	1	2	3	4
Polar coordinates	(5, 60°)	(4, 90°)	(3, 120°)	(2, 150°)	(1, 180°)



(a) Mark the locations of the snake on a polar coordinate plane.

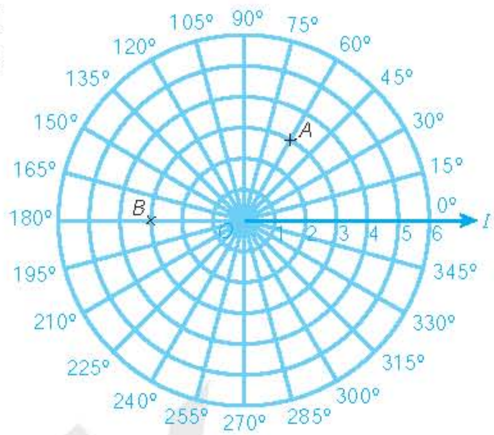
(b) Which quantity, the radius vector or the vectorial angle, should be used to determine whether the snake is approaching the cave? According to the quantity chosen, is the snake approaching the cave?

16. In the figure, the polar coordinates of A and B are (3, 60°) and (3, 180°) respectively. Draw each of the following triangles on a polar coordinate plane.

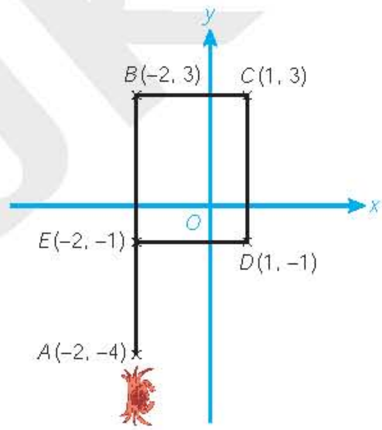
(a) Isosceles triangle ABC

(b) Equilateral triangle ABD

[Hint: A protractor can be used.]



17. In the figure, a crab starts from A and moves along AB, BC, CD and DE to reach E. Find the total distance travelled by the crab.



18. The following are the coordinates of the vertices of polygons. Draw each polygon on a rectangular coordinate plane and find its area.

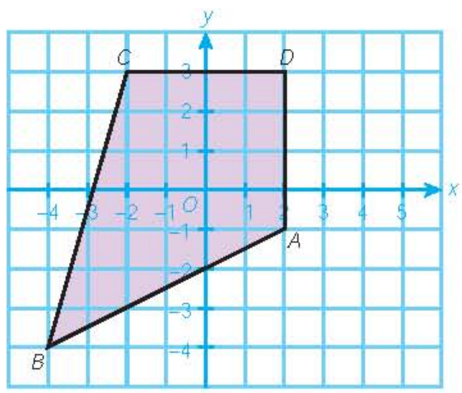
(a) $A(-3, 0)$, $B(-3, 4)$, $C(2, 2)$

(b) $P(4, 0)$, $Q(4, 5)$, $R(2, 5)$

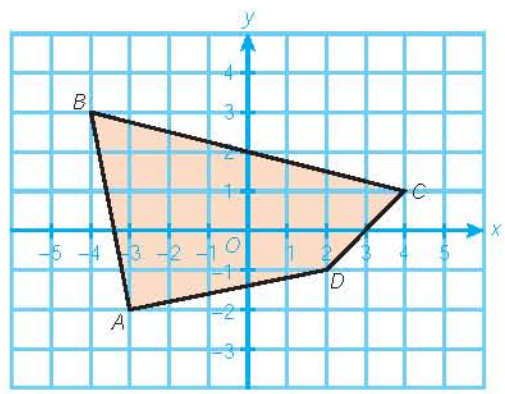
(c) $A(-2, -1)$, $B(2, -1)$, $C(2, 3)$, $D(-4, 3)$

19. Find the area of the shaded region in each of the following figures.

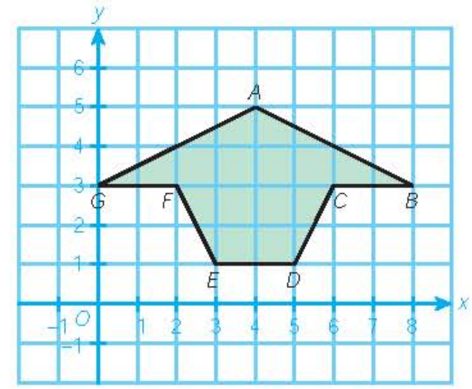
(a)



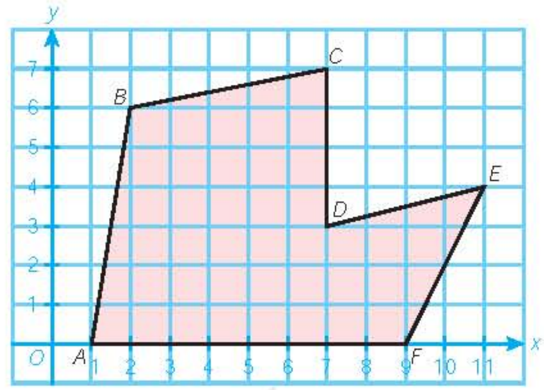
(b)



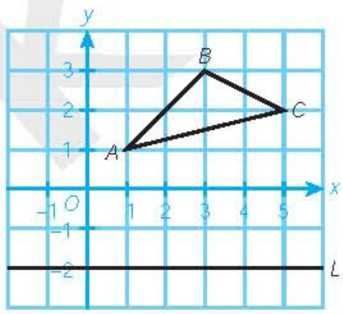
(c)



(d)



20. In the figure, ΔABC is reflected along straight line L to obtain the image $\Delta A_1B_1C_1$. Then $\Delta A_1B_1C_1$ is rotated clockwise about the origin through 90° to obtain the image $\Delta A_2B_2C_2$.

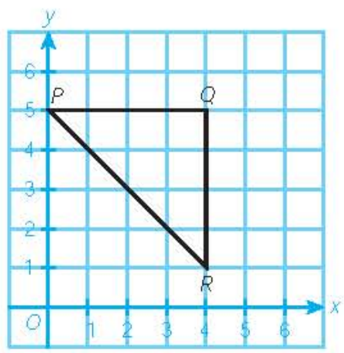


(a) Draw ΔABC, $\Delta A_1B_1C_1$ and $\Delta A_2B_2C_2$ on the same rectangular coordinate plane.

(b) Find the coordinates of the vertices of image $\Delta A_1B_1C_1$.

(c) Find the coordinates of the vertices of image $\Delta A_2B_2C_2$.

21. In the figure, ΔPQR is rotated anti-clockwise about the origin through 90° to obtain the image $\Delta P_1Q_1R_1$. Then $\Delta P_1Q_1R_1$ is rotated about the origin through 180° to obtain the image $\Delta P_2Q_2R_2$.



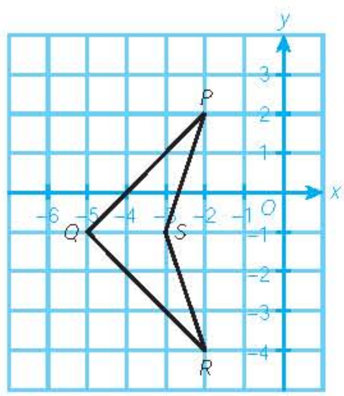
(a) Draw ΔPQR, $\Delta P_1Q_1R_1$ and $\Delta P_2Q_2R_2$ on the same rectangular coordinate plane.

(b) **(i)** Find the coordinates of the vertices of image $\Delta P_1Q_1R_1$.

(ii) Find the coordinates of the vertices of image $\Delta P_2Q_2R_2$.

(c) If ΔPQR changes to $\Delta P_2Q_2R_2$ after a single transformation, describe the transformation.

22. In the figure, quadrilateral $PQRS$ is rotated about the origin through 180° to obtain the image $P_1Q_1R_1S_1$. Then $P_1Q_1R_1S_1$ is reflected along the x-axis to obtain the image $P_2Q_2R_2S_2$.



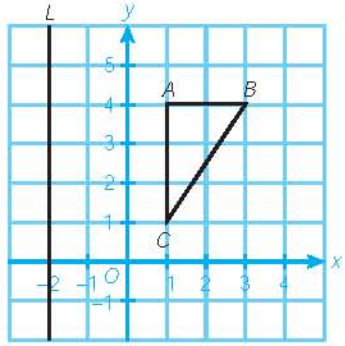
(a) Draw $PQRS$, $P_1Q_1R_1S_1$ and $P_2Q_2R_2S_2$ on the same rectangular coordinate plane.

(b) **(i)** Find the coordinates of the vertices of image $P_1Q_1R_1S_1$.

(ii) Find the coordinates of the vertices of image $P_2Q_2R_2S_2$.

(c) If $PQRS$ changes to $P_2Q_2R_2S_2$ after a single transformation, describe the transformation.

23. In the figure, ΔABC is reflected along straight line L to obtain the image $\Delta A_1B_1C_1$, and then translated 4 units downwards to obtain the image $\Delta A_2B_2C_2$. $\Delta A_2B_2C_2$ is then rotated clockwise about the origin through 90° to obtain the image $\Delta A_3B_3C_3$.

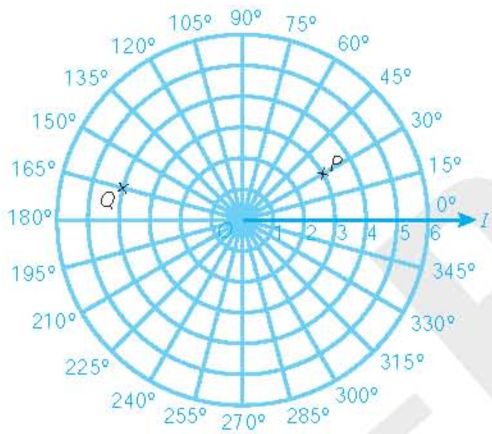


(a) Draw ΔABC, $\Delta A_1B_1C_1$, $\Delta A_2B_2C_2$ and $\Delta A_3B_3C_3$ on the same rectangular coordinate plane.

(b) Do ΔABC and $\Delta A_3B_3C_3$ have the property of reflectional symmetry? If they do, draw the axis of symmetry.

MC Question

24. In the figure, $\angle POQ =$



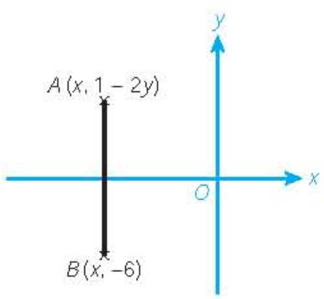
A. 135°.

B. 165°.

C. 195°.

D. 225°.

25. Given that the distance between $A(x, 1 - 2y)$ and $B(x, -6)$ is 9 units, find the value of y.



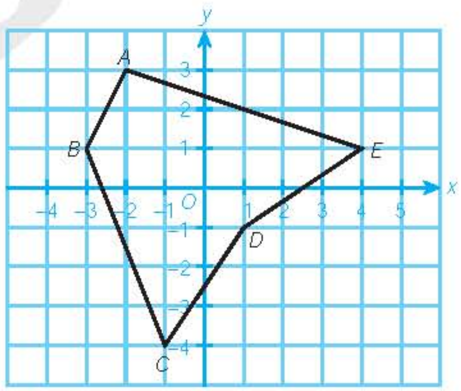
A. -1

B. -2

C. -3

D. -4

26. In the figure, find the area of pentagon $ABCDE$.



A. 10 square units

B. 15 square units

C. 22 square units

D. 28 square units

27. $P(-2, -2)$ is translated 5 units to the left and 7 units upwards to obtain the image P'. Find the coordinates of P'.

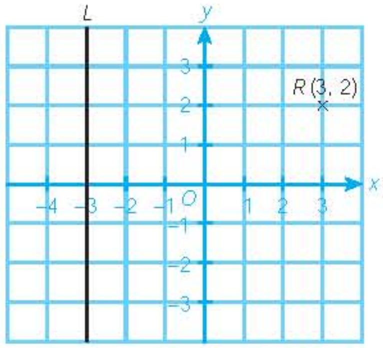
A. $(-7, -9)$

B. $(-7, 5)$

C. $(3, 5)$

D. $(3, 9)$

28. In the figure, $R(3, 2)$ is reflected along straight line L to obtain the image R'. Find the coordinates of R'.



A. $(0, 2)$
B. $(-3, 2)$
C. $(-8, 2)$
D. $(-9, 2)$

☐

29. $S(-5, 1)$ is rotated clockwise about the origin through 90° and then reflected along the y-axis to obtain the image S'. Find the coordinates of S'.

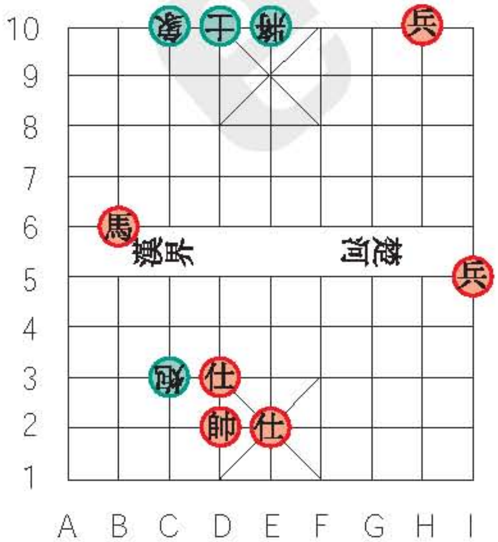
A. $(5, 1)$
B. $(1, -5)$
C. $(-1, 5)$
D. $(-5, -1)$

☐

Problem-solving and Exploring

Hint for the Title Page Question

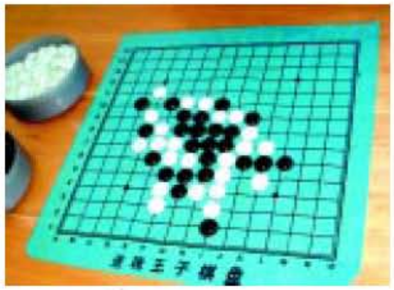
In the following, describe the path of the piece 馬 to catch the piece 將.



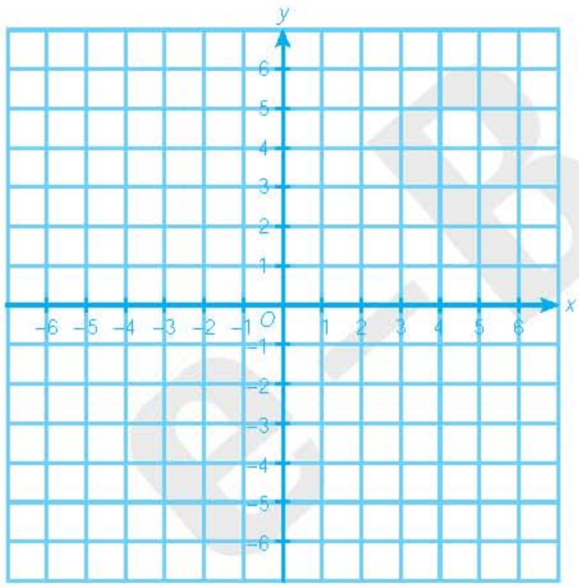
Additional Question

1. The game *gobang* is modified from the traditional Chinese gobang.

   Materials needed: Two rectangular coordinate planes on which the same whole numbers are marked (to be used as checkerboards) and a piece of white paper (to be used to record the coordinates of the pieces) as shown below.



Record sheet	
Player A	Player B



Number of players: 2

Rules: **i.** Each player takes a rectangular coordinate plane. One uses ‘X’ as pieces while the other uses ‘O’ as pieces.

ii. Each player marks the coordinates of his/her pieces on the record sheet in turn and the positions of the pieces on his/her own checkerboard.

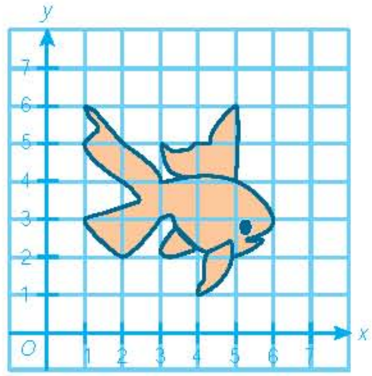
iii. The one who can arrange 5 of his/her pieces in a line wins. The records of the winner on the record sheet must be correct; otherwise, the other player wins. In case of disagreements, both players are allowed to check the record sheets and the corresponding checkerboards.

gobang 五子棋

2. When we stand in front of a magic mirror, our images will be distorted. In fact, we can bend a rectangular coordinate plane to simulate the effect of a magic mirror. The method is as follows:

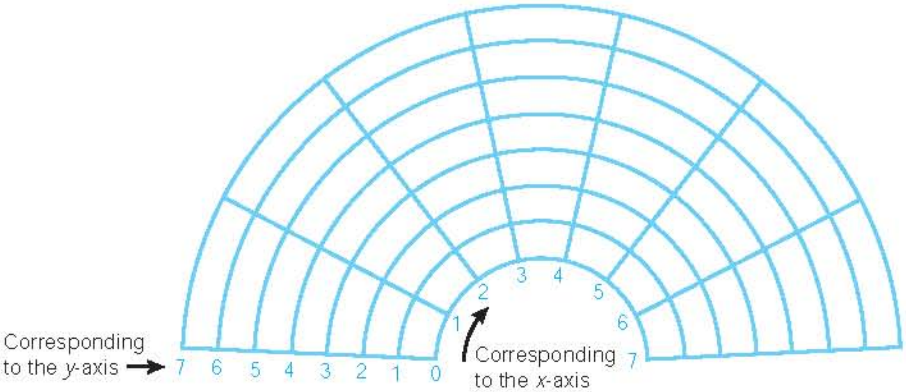
Step 1

Draw a rectangular coordinate plane on a graph paper, and draw a picture on it.



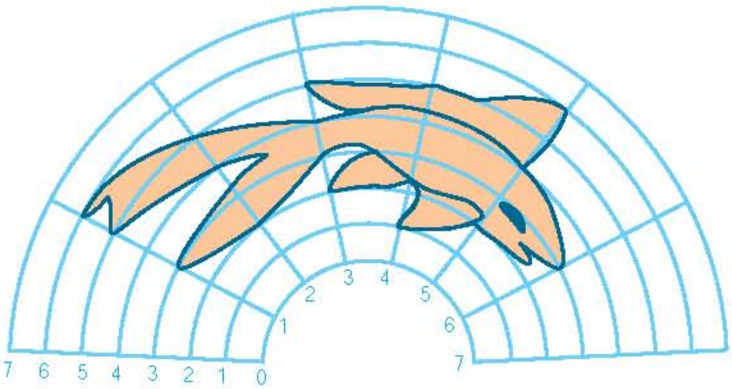
Step 2

Draw some equidistant arcs to form another coordinate plane.



Step 3

Transfer all points on the rectangular coordinate plane to the corresponding coordinates on this coordinate plane.



Similarly, draw the image of your favourite figure on a magic mirror.